An Empirical Differential Game for Sustainable Forest Management*

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Abstract

We model the role of the world's forests as a major carbon sink and consider the impact that forest depletion has on the accumulation of CO_2 in the atmosphere. Two types of agents are considered: forest owners who exploit the forest and draw economic revenues in the form of timber and agricultural use of deforested land; and a non-forest-owner group who pollutes and suffers the negative externality of having a decreasing forest stock. We retrieve the cooperative solution for this game and show the cases in which cooperation enables a partial reduction in the negative externality. We analyze when it is jointly profitable to abate emissions, when it is profitable to reduce net deforestation, and when it is optimal to do both (abate and reduce net deforestation). It is shown how the cooperative solution can be sustained by means of a time-consistent payment mechanism.

Key Words: game theory, dynamic games, optimal control, deforestation, time consistency, forest management, emissions.

1 Introduction

The world's forests cover nearly one-third of the earth's surface, but are decreasing at an alarming rate, with an area equivalent to the size of Costa Rica being

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deforested every year (FAO, 2010). World deforestation has become an issue of great international environmental concern for a number of reasons: First, the world's forests have an ecological value as carbon sinks. Second, forests host much of the world's biodiversity. Third, forests protect land and water resources and help prevent land erosion and desertification. In this paper we concentrate mainly on the role of forests as carbon sinks, although the framework used here could be extended to include the other two aspects.

We view forests as providers of competing economic and environmental goods. While forest logging brings economic revenues from both timber and agriculture on deforested land in the short run (FAO, 2006), excessive logging can exacerbate the problem of greenhouse gases (GHGs) accumulating in the long run. We build a model that accounts for GHG accumulation in the atmosphere in terms of anthropogenic emissions and carbon sequestration by the world's forests. The framework used allows us to (i) evaluate the impact that forest depletion has on atmospheric GHG accumulation through the so-called *reduced-carbon-sequestration effect,* which states that a tree that is cut cannot grow and hence cannot sequester carbon; and (ii) compare the short-term rewards of high emissions and intensive deforestation policies with their long-term costs due to excessive GHG accumulation and forest depletion.

There is a significant dynamic-games literature dealing with the role of excessive GHG accumulation (see, e.g., the early papers of van der Ploeg and de Zeeuw (1992), Long (1992), Dockner and Long (1993) and the literature review by Jørgensen et al. (2010)). In this literature, emissions are a control variable and the issue is to determine the optimal emissions rate so as to reduce the environmental damage coming from the excessive accumulation of GHGs. Typically, these models concentrate on the difficulty of coordinating optimal emission levels and treat carbon sequestration as exogenously given. A first contribution of this paper to this literature is in explicitly accounting for the role of forests as a carbon sink.

There is also a literature that deals with forest depletion using a dynamic-game approach (e.g., van Soest and Lensink (2000), Fredj et al. (2004, 2006), Martín-Herrán and Tidball (2005) and Martín-Herrán et al. (2006)). In these articles the players are forest owners, who exploit their asset to obtain economic revenues, and a donor community, or an environmentally-aware player, that is willing to compensate forest owners who engage in preservation efforts of the resource.

We develop in this paper a model that merges these two strands of the literature. On the one hand forest owners exploit (and eventually deplete) the forest. Their actions have an environmental impact on the atmospheric accumulation of GHGs. On the other, the non-forest-owner group derives utility from production (i.e., emissions) and disutility from the accumulation of GHGs in the atmosphere. In this setting, it is this disutility they experience that may eventually turn them into donors to preserve the forest as a carbon sink. This modelling framework allows us to capture both the high opportunity cost of reducing deforestation and the negative economic externality that forest owners inflict on non-owners as a consequence of their deforestation policy. Unlike the other aforementioned papers we do not focus solely on forest conservation but also on its impact on GHG accumulation. Non-owners are to decide on their optimal level of emissions.

The parameters of the model are empirically estimated, a rarity in this area. We believe that this constitutes a valuable contribution to the literature and an interesting demonstration case for decision makers on how strategic interactions affect the evolution of both GHG accumulation and forest depletion. We determine the jointly optimal outcomes and compare them to the non-cooperative or business-as-usual counterparts. When the planning horizon is sufficiently long, then the cooperative solution is overall welfare improving. Cooperation partly reduces the negative externality and we analyze when it is profitable to abate emissions, when it is profitable to reduce net deforestation, and when it is optimal to do both (abate and reduce net deforestation). The results obtained show that it is preferable (cheaper) to invest in deforestation reduction than in emissions abatement when the perceived damages are low. However, as the environmental damage increases, it becomes optimal to combine emissions abatement with deforestation reduction.

The cooperative solution brings economic gains; however they are asymmetric: the non-forest-owner group gains while forest owners lose. This implies that any environmental agreement attempting to implement the cooperative solution will require monetary compensation from the agents who win (non-owners) to the agents who lose (forest owners). Necessarily, forest owners have to be compensated with an amount at least equal to the difference between the sum of their intertemporal cooperative and non-cooperative payoffs. However, this requirement is not enough. When designing an intertemporal compensation mechanism, it is of key importance to allocate the transfers in such a way that no player has an economic incentive to deviate from the cooperative agreement at any instant of time, i.e, that the agreement be time consistent. We show that a division of joint payoffs using a dynamic Nash-Bargaining Scheme yields time-consistent outcomes.

The remainder of the paper is organized as follows: The model used for the two types of agents is presented in Section 2. In Section 3, the non-cooperative optimal policies for each player are obtained. Then, in Section 4, we compute the optimal cooperative policies and compare them to their non-cooperative counterparts. A robustness analysis is also performed. Section 5 is devoted to analysing the feasibility and dynamic stability of the cooperative solution. Finally, all the results obtained are summarized in Section 6.

2 The model

We consider two types of agents: forest owners and non-owners. Forest owners are environmentally unconcerned agents who only care about revenues from deforestation, that is, they do not consider the consequences of their deforestation policy on GHG accumulation. Conversely, non-owners get revenues from the production of economic goods. Their productive activity generates emissions and this non-forest-owner group does take into account the negative effects of their current emissions policies on GHG accumulation in the atmosphere. This way of modelling allows us to capture the negative externality that forest owners create on the non-forest-owner group through the so called *reduced-carbon-sequestration effect*.

2.1 The forest owners' problem

Forest owners maximize their discounted stream of net revenues. Forest revenues depend on their afforestation and deforestation rates, $A(t)$ and $D(t)$, respectively, as well as on the existing forest surface area $F(t)$, measured in hectares. Net revenues are discounted at rate r throughout a finite time horizon, given by time T. In the next section, we let the planning horizon vary and show how the optimisation results depend on the value of T.[1]

Net revenues include gross revenues $R(t)$, afforestation costs $\kappa_1 A(t)$ and deforestation costs $\kappa_2 D(t)$, where κ_1 and κ_2 are respectively the per-hectare afforestation and deforestation costs. The forest owners' objective is the following:

$$\max_{A(t),D(t)} \int_0^T e^{-rt} \left[R(t) - \kappa_1 A(t) - \kappa_2 D(t)\right] dt, \tag{1}$$

where $A(t) \in [0, A_{\max}]$ and $D(t) \in [0, D_{\max}]$. The upper bounds for afforestation ($A_{\max}$) and deforestation ($D_{\max}$) reflect the idea that there is a physical limit, in the short term, to afforestation and that deforestation is subject to a regulation that allows for it within certain limits. The value of $D_{\max}$ is set to fit the observed world deforestation figures provided by the FAO (2006). The definitions of all parameters, their values and their sources are provided in Appendix A.

We assume that the evolution over time of the forest area can be well approximated by the following linear differential equation:

$$\dot{F}(t) = A(t) + \eta F(t) - D(t), \quad F(0) = F_0, \tag{2}$$

where η is a positive parameter, and F_0 is the initial forest world's surface area in 2005 (FAO, 2006), i.e., nearly four billion hectares. Equation (2) is an extension of van Soest and Lensink (2000) and Fredj et al. (2006), where $A = \eta = 0$ in the first and $A = 0$ in the second. Note that the linear specification in (2) approximates reasonably well forest expansion within a large interval around the current world forest area $F(0)$.

Forest owners obtain revenues from selling timber and agricultural products. Denote by $q(t)$ the quantity of timber put on the market at time t, and let the price $p(t)$ be given by the following inverse demand function:

$$p(t) = \bar{p} - \theta q(t), \tag{3}$$

[1] As will be seen, in order to achieve and sustain cooperation, some sort of compensating transfers are required. The proposed transfer scheme makes sense for a finite planning horizon, but would be more difficult to justify for an infinite one.

where $\bar{p}$ is the choke price that makes the demand equal to zero, and θ is the average price elasticity of demand. The values of parameters $\bar{p}$ and θ have been calibrated using data given by the FAO on timber prices and quantities.

The quantity $q(t)$ comes from two different sources, namely, clear-felling and selective logging, and is given by

$$q(t) = yD(t) + y\gamma\delta F(t), \tag{4}$$

where $yD(t)$ is the amount of wood retrieved from clear-felling an area $D(t)$ and the product $y\gamma\delta F(t)$ stands for the total selective-logging yield, which is lower (in per-hectare terms) than the yield obtained through clear-felling. Parameter y denotes the per-hectare timber yield and is typically measured in stems per hectare or cubic meters of timber per hectare. The FAO (2006) provides an estimate for this parameter. Clear-felling an area $D(t)$ reduces the total forest size by the same amount. However, unlike deforestation, selective logging is assumed to have no impact on total forest land. According to the FAO, "[selective logging]*...is not necessarily destructive and can be done with low impact on the remaining forests, if the proper techniques are applied*".[2] Clearly, for selective logging to have a negligible environmental impact, its per-hectare yield per unit of area must be much lower than clear-felling. This lower yield is accounted for by parameter γ ($\gamma << 1$). Finally, according to the FAO (2006), roughly one-third of the world's forests are used primarily for the production of wood and non-wood forest products. Parameter δ takes into account the fact that only a fraction of the world's forests are actually being exploited.[3]

Agriculture revenues are equal to the prices times the yields of the different crops grown. For simplicity, we suppose that forest owners grow a single agricultural good, which we model as a composite good made of four representative crops that are commonly related to deforestation processes. This good is sold in international markets at a given price p_A.[4] The total yield at time t depends on the size of the available (previously deforested) land, given by $\bar{F} - F(t)$, where $\bar{F}$ stands for the maximum size or carrying capacity of the forest, and on the soil productivity $x(t)$. As in Andrés-Domenech et al. (2011)–see also van Soest and Lensink (2000) for a simpler version–we model $x(t)$ as follows:

$$x(t) = \bar{x} + \alpha(t)D(t) - \beta\frac{\bar{F} - F(t)}{\bar{F}}. \tag{5}$$

The above expression of the total productivity of land $x(t)$ is the sum of three terms. The first is a constant productivity term $\bar{x}$ that measures the average yield in tons of crop per hectare of land for a representative agricultural good. The second term, $\alpha(t)D(t)$, captures the idea that newly deforested land $D(t)$ is

[2]Source: http://www.fao.org/forestry/news/48681/en/

[3]The equation that we used for q is a small variation of the one presented by van Soest and Lensink (2000). In their case γ and δ are assumed equal to one. Here we follow the more comprehensive specification used by Andrés-Domenech et al. (2011).

[4]The price p_A is constant, unlike $p(t)$, due to the fact that agricultural production on deforested land represents only a fraction of the world's total agricultural land.

more productive. Variable $\alpha(t)$ measures the increase in the *total* average per-hectare production resulting from deforesting an area $D(t)$. The third term, $-\beta\frac{\bar{F}-F(t)}{\bar{F}}$, accounts for the positive externality that forests generate on nearby agricultural land. Forests are seen as a source of rain and a protective element for agricultural land. Parameter β measures the decrease (increase) in soil quality, and therefore, in agricultural productivity caused by forest depletion (expansion). The productivity increase of newly deforested land is given by

$$\alpha(t) = \frac{\psi\bar{x}}{\bar{F} - F(t)}. \tag{6}$$

Newly deforested land is more productive and parameter ψ measures the factor by which productivity is increased. However, this extra productivity needs to be normalised for all agricultural land. We divide the extra yield, $\psi\bar{x}$, by the total agricultural surface area, $\bar{F} - F(t)$, otherwise the term $\alpha(t)D(t)$ in equation (5) would overestimate the real impact that deforesting an area $D(t)$ has.[5]

Putting together the revenues from timber sales and agricultural products, we get the following expression for gross revenue:

$$R(t) = p(t)q(t) + p_A x(t)\left(\bar{F} - F(t)\right). \tag{7}$$

To recapitulate, forest owners maximize their net discounted economic revenues (1) with respect to their deforestation and afforestation efforts, $D(t)$ and $A(t)$, respectively, subject to the forest dynamics in (2).

2.2 The non-owners' problem

Non-owners optimize a two-part objective function. The first part consists of a short-run gain derived from producing and consuming economic goods. The production of these goods generates pollution as a by-product and this pollution affects their utility. For simplicity, we suppose that the carbon intensity of the economy is constant. Hence, *ceteris paribus*, producing more goods is equivalent to emitting more.[6] Denote by $E(t)$ the GHGs emissions by the non-forest-owner group; and by the concave increasing function $G(E)$ the payoff generated in terms of goods production. We adopt the following functional form:

$$G(E(t)) = aE(t) - \frac{1}{2}bE^2(t), \tag{8}$$

where parameters a and b are positive and have been fixed in order to ensure that $G'(E) > 0$ for the relevant range of emissions. This specification is similar

[5] Agricultural revenues are obtained by multiplying the productivity (5) by total agricultural land. Hence equation (5) has to account for average *per-hectare* productivity measured in tons of crop per hectare. For this reason, the term $\alpha(t)D(t)$ cannot be understood as the extra productivity of newly deforested land, but rather as the normalised productivity increase that *newly* deforested land has on *total* agricultural land.

[6] One could think of a more refined formulation, where the economy's carbon intensity can adjust, and where production increases can be compatible with constant emissions levels or even decreases.

to the one proposed in, e.g., Dockner and Long (1993) and Breton et al. (2005), with the only difference being that we have included parameter b to calibrate $G(E(t))$ at the current world-level GDP.

The second term in the objective of the non-forest-owner group represents an economic loss or damage related to the accumulation of emissions in the atmosphere. Denote by $S(t)$ the stock of GHGs (e.g., stock of CO_2) in the atmosphere at a given time t. According to the IPCC (2007), increases in the atmospheric concentration of GHGs result in seawater levels rising, temperatures increasing and seawater acidification. These processes are all related to economic and environmental damage. We assume that the damage cost is given by a strictly convex and increasing function $L(S)$. Although we acknowledge the existence of thresholds, extreme events and jumps in the damage,[7] our formulation, which is very common in the literature (see, e.g., Benchekroun and Long (2002), Dockner and Long (1993), van der Ploeg and De Zeeuw (1992), Breton et al. (2006)), smooths the impacts of such phenomena rather than dealing with them explicitly. Needless to say, accounting properly for non linearities and threshold effects in the damage cost would lead to a model of much greater complexity.

That being said, for a specific function to qualify as a good candidate to model such damages we can think of yet another necessary requirement: greenhouse gases, and most particularly CO_2, have always been present in the atmosphere and represent a basic element for the existence and development of life (e.g., plants). It is clear that it is not the presence, but the excessive accumulation of atmospheric GHGs that poses the problem. We adopt the following specification of $L(S)$ to capture all these elements in a simple way:

$$L(S(t)) = c(S(t) - \underline{S})^2, \tag{9}$$

where $\underline{S}$ is a natural threshold, beyond which economic and environmental damage is considered excessive. In practical terms, choosing a reasonable value for $\underline{S}$–given the above specification–amounts to choosing a level of atmospheric GHGs for which there is no perceived damage. We identify $\underline{S}$ with the preindustrial level of GHGs (see, e.g., Bahn et al. (2008)).

Taking into account the gain function $G(E)$ and the damage function $L(S)$, we obtain the following objective functional, which is maximized by the non-forest-owner group:

$$\int_0^T e^{-rt} \left[G(E(t)) - L(S(t))\right] dt - \phi\left(S(T)\right) e^{-rT}, \tag{10}$$

where r is the market discount rate (the same as for forest owners), and $\phi(S(T))$ is a salvage value.

[7]For instance, a small increase in the atmospheric concentration of GHGs can bring a quantitatively different damage but may also trigger qualitatively different damages (e.g., massive ice-cap melting, dissolution of coral reefs as a result of extreme oceanic acidification, etc.).

Non-owners are modelled as forward-looking agents who consider the long-term impact of their decisions. The stock of emissions accumulates slowly and then has a long-term impact on non-owners' payoffs. Therefore, it is sensible to have a scrap-value function somehow related to the stock of emissions at the terminal date of the planning horizon. Such a salvage-value function can be generically written as $\phi(S(T))$. It is reasonable to think that, whatever the GHG stock at the terminal date, it will strongly impact future payoffs due to the long-term persistence of greenhouse gases in the atmosphere. One could think of a more sophisticated scrap-value function that also depends on the final forest stock or on the emissions policy followed after the terminal date, or even define the scrap-value function as an identical problem to the one presented above in equation (10). Because we want to keep the model parsimonuous, and because we want to be able to say something that is irrespective of what policies are chosen after the terminal date, we have chosen the following formulation for $\phi(S(T))$, which depends on the terminal stock of greenhouse gases alone:

$$\phi\left(S(T)\right) = \int_{T}^{2T} e^{-r(s-T)}\, L(S(T))\, ds. \tag{11}$$

Although the salvage function in (11) is simple, it satisfies the following intuitive requirements: (i) It reflects the idea that the terminal stock of GHGs matters and has an impact on future payoffs; (ii) it is easy to compute and does not depend on (potentially) unknown future policies; (iii) it keeps discounting in a natural way the cost of future environmental damages; and (iv) the time span considered for the scrap value function is related to the planning horizon. In fact, if the planning horizon chosen is short, then the weight given for future environmental damages will likely be small as well and vice versa.

Non-owners maximize their payoffs in (10) by adjusting their emissions, and their decision has an impact on the state of the system. Emissions, in our model, are assumed to be exclusively anthropogenic and are given entirely by the emissions of the non-forest-owner group. By this we do not mean that forest owners do not emit but rather that their contribution to global emissions is negligible. The dynamics of the emissions rate $E(t)$ is then given by

$$\dot{E}(t) = V(t)E(t), \quad E(t) \geq 0,\ E(0) = E_0. \tag{12}$$

The dynamics of emissions in equation (12) can also be written in a more familiar way, that is,

$$\frac{\dot{E}(t)}{E(t)} = V(t),$$

where $V(t)$ denotes the instantaneous speed of emissions variation. For the sake of realism $V(t)$ has been modelled as a bounded control variable (i.e., $V_{\min} \leq V(t) \leq V_{\max}$), with $V_{\min} < 0$ and $V_{\max} > 0$. In the literature, it is more common to see emissions as a flow variable. Our modelling of the emissions allows us to better account for the inertia of the productive and economic systems. Indeed, emissions take time to adjust and the upper and lower bounds on $V(t)$ simply reflect this idea that emissions cannot be increased or decreased at whatever rate. One can think of these bounds as being given by the existence of technical, economic and/or political constraints.

The evolution of the stock of greenhouse gases in the atmosphere depends on emissions and on carbon sequestration by the world's forests and oceans. Forests worldwide sequester carbon as they grow, and according to the IPCC (2000) and FAO (2006), approximately half of the dry weight of forest biomass is carbon. To model carbon sequestration by forests, one could measure the variation in the total forest biomass; however, this would present two main difficulties. First, the variation in total carbon biomass is difficult to measure. And second, measuring carbon sequestration through the variation in forest biomass underestimates the total carbon sequestration since timber captures are neglected. To overcome this problem, we make the simplifying assumption that forest owners manage a representative forest whose trees grow–volume wise–at an average and constant rate. Having a representative forest whose growth rate is constant allows us to express carbon sequestration as a linear function of forest area alone (i.e., carbon sequestered per hectare of forest land and per unit of time). The advantage of having carbon sequestration in terms of forest area rather than in terms of biomass variation is that one can easily consider timber captures, while gaining a tractable and understandable way to measure carbon sequestration.

Further, note that by measuring carbon sequestration as a function of the forests' surface area, one can account for the so called *reduced-carbon sequestration effect,* which is based on the simple principle that a tree that is cut cannot grow (i.e., cannot sequester carbon). Thus, it is straightforward to see that deforestation has a negative impact on carbon sequestration due to the reduction in forest area that it induces. Expression (13) below captures the dynamics of the atmospheric concentration of carbon in terms of the forest stock, where parameter φ reflects the amount of carbon sequestered per hectare of forest and per unit of time.

We also consider the oceans as a second type of carbon sink. Denote by W the amount of carbon that they sequester. The carbon uptake by the oceans has remained relatively stable during the last few years, and for this reason, W has been assumed constant for simplicity's sake even if there exist small year-to-year variations due to El Niño effects (Le Quéré et al. (2009)). The evolution of the stock of pollution is then given by the following differential equation:

$$\dot{S}(t) = E(t) - \varphi F(t) - W, \quad S\,(t) \geq 0, \quad S(0) = S_0. \tag{13}$$

To wrap up, the non-owners maximize their payoff in (10) adjusting the instantaneous variation of emissions $V(t)$, subject to equations (12) and (13) and given the fact that the solution to (2) is inherited from the forest owners' problem.

3 Individual optimization

In this section we characterize the optimal strategies of the two players when they act independently. As the forest owners' payoffs do not depend on emissions or on GHG accumulation, their payoffs are independent of the action of non-owners. On the other hand, non-owners' payoffs are affected by the forest owners' decisions through the evolution of the forest stock. In this setting, where there is a one-way interaction, Nash and Stackelberg equilibria coincide. Further, open-loop and feedback-information structures yield the same result. Given this, we can first solve the economic problem of the forest owners, and next, optimize for the non-forest-owner group, taking the evolution in the forest stock as given.

In the rest of the paper we use $\mathcal{O}$ to denote the forest owners and $\ominus$ to refer to the non-owners.

3.1 Forest owners

Forest owners maximize their revenues in (1) subject to (2)-(7). The following proposition provides the optimal solution to their control problem (the superscript *nc* stands for noncooperation).

Proposition 1 *For the parameter domain defined in Appendix A, the optimal control, state and co-state variables are given by*[8]

$$A^{nc}(t) = 0, \quad D^{nc}(t) = D_{\max} \ \textit{for all}\, t \in [0,T],$$

$$F(t) = \left(F_0 - \frac{D_{\max}}{\eta}\right) e^{\eta t} + \frac{D_{\max}}{\eta}, \tag{14}$$

$$\lambda(t) = \frac{1}{\eta - r}\left[1 - e^{(\eta-r)(T-t)}\right]\{(2\theta y D_{\max} - \bar{p})y\gamma\delta + p_A(\bar{x} - 2\beta) + 2F(t)\left[\theta y^2\gamma^2\delta^2 + p_A\beta\frac{1}{\bar{\bar{F}}}\right]\Bigg\}. \tag{15}$$

Proof. See Appendix B. ■

The results show that the forest owners' optimal strategy consists in deforesting at maximum admissible level and not afforesting at all. As the problem is linear in the afforestation effort, and afforestation is a pure cost in our setting, then the optimal strategy is obviously to set $A(t)$ at its lowest admissible value, i.e., $A(t) = 0$. Further, the marginal revenue from agricultural activity is positive for all admissible values of $D(t)$, including $D_{\max}$. Therefore, there is an incentive to deforest at the maximum level. These results follow from the fact that, for our parameter domain, we have $\lambda(t) \leq 0$, for all t. Indeed: (i) the term $\frac{1}{\eta-r}\left[1 - e^{(\eta-r)(T-t)}\right]$ is always negative since $\frac{1}{\eta-r}$ and $\left[1 - e^{(\eta-r)(T-t)}\right]$ are of opposite signs, regardless of the values of η and r; and

[8]The second-order sufficient optimality conditions are satisfied for this and all the problems studied in this paper.

(ii) $(2\theta y D(t) - \bar{p}) y \gamma \delta + p_A(\bar{x} - 2\beta) > 0$, for all admissible values of $D(t)$, including $D_{\max}$. Deforestation is mainly driven by the revenues obtained from growing agricultural products on deforested land, rather than by the timber revenues that arise from deforestation itself. This is in line with other studies, e.g., Barbier and Rauscher (1994), Barbier and Burgess (2001) and FAO (2006), which suggested that deforestation for agricultural purposes is the main explanatory factor for forest depletion worldwide.

3.2 Non-owners

The non-owners maximize their payoff given by (10) and take into account the values of the three state variables, namely, forest area, F, emissions, E, and the stock of accumulated emissions in the atmosphere, S. The optimal solution depends on the length of the planning horizon and on the intertemporal discount rate. For the values of our parameters, the solution is constant ($V^{nc} = V_{\max}$) as long as the planning horizon (T) is less than approximately forty years, (i.e., $T \lesssim 40$).[9] The following proposition provides the optimal solution to the problem of non-forest owners and the optimal time paths for control and state variables in such a case.

Proposition 2 *For the parameter domain defined in Appendix A and $T \lesssim 40$, the optimal control and state variables are given by*

$$V(t) = V^{nc} = V_{\max} \ \text{for all} \ t \in [0, T],$$

$$E(t) = E_0 e^{V^{nc} t}, \tag{16}$$

$$S(t) = S_0 - \frac{\varphi}{\eta} t D_{\max} - \frac{E_0}{V^{nc}}(1 - e^{V^{nc} t}) + \frac{\varphi}{\eta}\left(F_0 - \frac{D_{\max}}{\eta}\right)(1 - e^{\eta t}). \tag{17}$$

Proof. See Appendix C. ■

As pointed out above, the optimal control V^{nc} depends on the planning horizon being considered. For a relatively short horizon, i.e., $T \lesssim 40$, the optimal solution is constant of the type $V^{nc} = V_{\max}$ all along. The solutions shown in Proposition 2 hold for as long as there is no switching time. For longer horizons, i.e., $T \gtrsim 40$, the optimal solution is to apply the control $V = V_{\max}$ for some time and then switch to a cleaner regime.[10] For much longer horizons (i.e., $T > 100$), it is possible that the optimal solution consists of switching not once but several times. In all cases, the different switching times and the number of switches depend on the value adopted for T. Denote by $\tilde{t}_V$ the optimal switching time. Then the optimal solution for $40 \lesssim T \leq 100$ can be summarized as follows:

$$V(t) = \begin{cases} V_{\max}, & \text{for } t \leq \tilde{t}_V \\ V_{\min}, & \text{for } t > \tilde{t}_V. \end{cases}$$

[9]The determination of the exact planning horizon beyond which Proposition 2 does not hold depends on the intertemporal discount rate. As we will see, for every value of the discount rate, we can obtain the maximum value of T for which Proposition 2 holds.

[10]It can be shown that singular arcs are not possible.

In Appendix D we have solved the problem for the case where there is only one switching time, and characterized the first-order conditions that apply in that case. Retrieving the actual switching time, however, represents a challenge. This is mainly due to the change in the evolution of the state and co-state variables as a consequence of changes in the switching time itself. The first-order conditions before and after the switch will only be satisfied if the exact switching time is chosen. This poses a problem in determining the actual switching time since one has to try an infinite number of possibilities, and the first-order conditions will only be satisfied if the exact one is chosen.

To overcome this problem we have developed an algorithm to obtain the optimal switching time, approximated to the integer value at which it is best to switch. The proposed algorithm consists of evaluating the sum of the payoffs for all possible scenarios (i.e., all possible switching times). From them, we then select the integer time for which the shifting regime (from $V_{\max}$ to $V_{\min}$) yields the greatest payoffs. A sketch of the algorithm can be found in Table 1.

Table 1: Sketch of algorithm used to compute the optimal switching time $\tilde{t_V}$

Fix the length of the planning horizon (T) and the discount rate (r)
for all possible integer switching times (t_V) **do**
  Payoff(t_V) = Discounted sum of payoffs before switch t_V
    + Discounted sum of payoffs after switch t_V
    + Scrap-value function
end
Select the t_V whose Payoff is greatest

Suppose for instance that our planning horizon and discount rate were fixed at, e.g., $T = 50$, $r = 0.02$. Figure 1 gives the payoffs of the non-forest-owner group in the y-axis for each possible switching time (x-axis). We observe, for this particular case, that switching from $V_{\max}$ to $V_{\min}$ after $\tilde{t_V}$ (where $\tilde{t_V} = 17$ years) is the best course of action.

We can generalize the algorithm presented in Table 1 and let the planning horizon T, vary while keeping the discount rate r constant. In so doing we obtain the best switching time for each different planning horizon.

Figure 2 gives the optimal switching time for each possible planning horizon T. To better understand this figure, it is important to distinguish between three elements, namely,

T : the planning horizon;
T^s : the minimum planning horizon for a switch to take place;
$\tilde{t_V}$: the actual switching time.

The 45-degree diagonal indicates that no switch is applicable. The shortest planning horizon for which there is a switch, T^s,is the first element of the curve off the diagonal. Figure 2 illustrates the fact that it pays to emit more in the short run. It also shows that for longer planning horizons it is comparatively

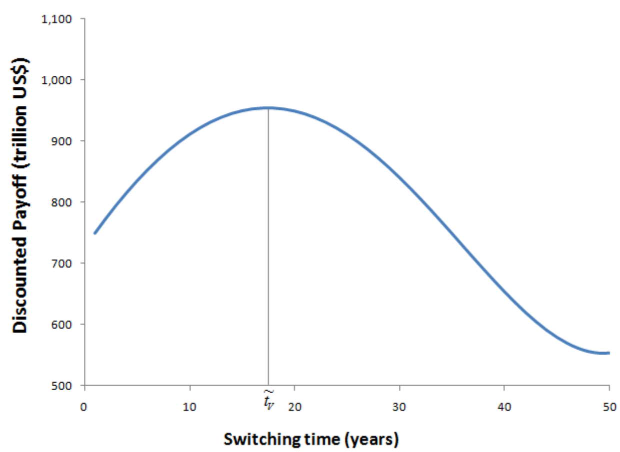

Figure 1: Payoffs as a function of the switching time

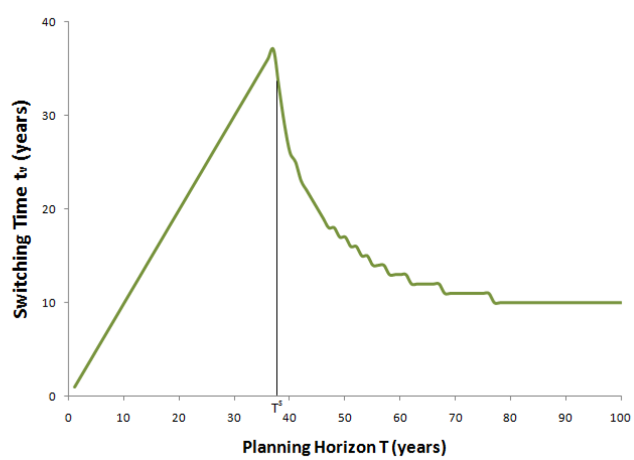

Figure 2: Optimal switching time for every planning horizon

less attractive to apply $V_{\max}$. This result is related to the existence of the non-linear damage function $L(S)$, by which the environmental damage increases when GHGs accumulate due to excessive emissions.

In Figure 2, $T^s = 38$. This means that there is no switch if $T < 38$, and that there will be one if $T \geq T^s$. As mentioned before, T^s and $\tilde{t_V}$ do not coincide, even when $T = T^s$. Put differently, if the planning horizon is long enough the non-forest-owner group recognizes the need to switch to a cleaner regime, but the switch will take place some time before the terminal date. Note that the pair $\left(T = 50,\ \tilde{t_V} = 17\right)$, which we obtained in Figure 1, is now just one point of the curve displayed in Figure 2.

We can further generalize our algorithm for any value of r. In the previous two figures, r was set equal to 0.02 (2%). The previous results are compared with two other alternative scenarios, $r = 1\%$ and $r = 3\%$, in Figure 3.

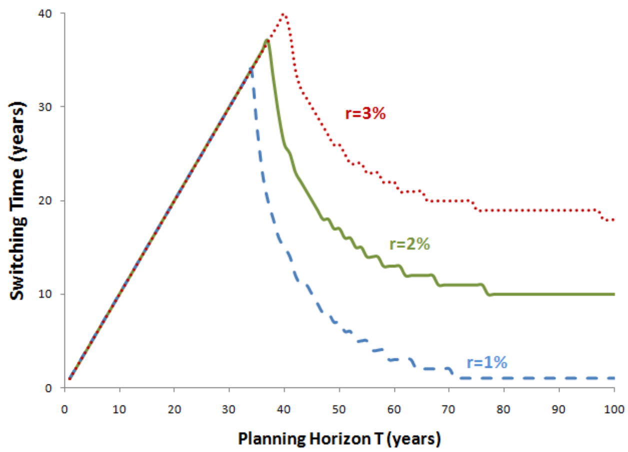

Figure 3: Impact of the discount rate on the switching time

Figure 3 conveys a dual message: First, when the discount rate is lower, the non-owners internalize the negative externality earlier, due to the accumulation of GHGsin the atmosphere. This can be inferred from the fact that T^s is lower for lower discount rates. In particular, we have that $T^s = 35$ if $r = 1\%$; $T^s = 38$ if $r = 2\%$; and $T^s = 41$ if $r = 3\%$. Second, the longer the time horizon used, the earlier the switch, i.e., the three curves are downward sloped.

To summarize, it is optimal for the non-owners to increase emissions if $T \lesssim 40$. If $T \gtrsim 40$, it is better to switch to a cleaner regime ($V = V_{\min}$) at some time $\tilde{t_V}$. The optimal time for the switch directly depends on the planning horizon and the discount rate used. A simple folk conjecture says that the longer the planning horizon and/or the smaller the intertemporal discount rate, the sooner this switch will arrive. This is related to the damage function $L(S)$, which yields greater (cumulative) losses for lower discount rates and longer planning horizons.

It has been shown how to determine the switching time. To put things into perspective, one can compare in Figure 4 the difference between the payoff with the optimal solution with switching time, $\pi(\hat{V})$, versus the payoffs $\pi(V_{\max})$ and $\pi(V_{\min})$ obtained by applying the constant (and sub-optimal) solutions $V = V_{\max} \forall t \in [0, T]$ and $V = V_{\min} \forall t \in [0, T]$ respectively. The value of $\pi(\hat{V})$ in Figure 4 is obtained by computing expression (10) for $r = 2\%$ along the optimal path for $E(t)$ and $S(t)$. For $T < 38$, $\pi(\hat{V})$ and $\pi(V_{\max})$ coincide. If $T \geq 38$, the curve $\pi(\hat{V})$ is obtained by applying a switch.

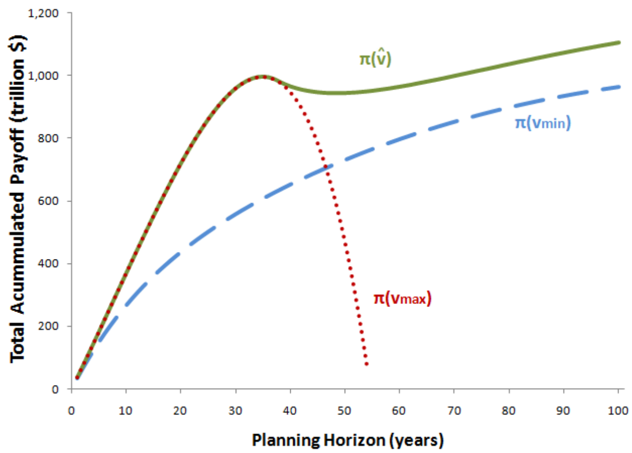

Figure 4: Comparing $\widehat{V}$ with $V_{\min}$ and $V_{\max}$

So far we have analyzed the optimal emissions policy. It is also important to analyze the sign of the shadow price of the forest stock, λ_F. This shadow price is positive regardless of the time horizon and discount rate considered. The positive sign of the co-state λ_F is directly related to the ability that forests have to sequester carbon. Since the increase in forest area is directly related to the enhancement of carbon sequestration (see expression (13)); then, regardless of the value of F and S,[11] marginally increasing the forest area implies marginal reductions in S, meaning smaller environmental losses (see expression (9)). This is a qualitative aspect.

At the same time, we have seen that the importance of reducing emissions is directly related to the length of the planning horizon and inversely related to the discount rate. Likewise, the marginal value that non-owners attach to an additional hectare of forest is greater when the planning horizon is longer and the discount rate is lower. This is a more quantitative aspect.

In short, unlike forest owners, the non-owners are interested in increasing the total forest area, and this is reflected by the sign of λ_F. If we compare the different ways in which forest owners and non-owners evaluate an additional hectare of forest, it is clear that there exists an environmental externality. Recall that forests have at least two uses: (i) the provision of economic revenues; and (ii) carbon sequestration. These uses are competing and somewhat mutually excluding. Forest owners create a negative externality on non-owners with their net deforestation policy. Hence the question is: should this negative externality be corrected?

Given the existing property rights over the forest, and the fact that forest owners' payoffs are a decreasing function of the total forest area, reducing the net

[11] Clearly we are referring here to values of S above $\underline{S}$.

deforestation is harmful for forest owners. Therefore, the answer to this question depends on whether an additional unit of forest can generate an increase in the payoff of the non-owners, such that it more than compensates for the reduction in the forest owners' revenues when they apply a more environmentally friendly deforestation/afforestation policy. If that is the case, then it will be jointly optimal to correct the externality, or at least part of it. In the next section we compute joint payoffs to answer the question raised above. We also compare the cooperative scenario to the *status-quo* individual equilibrium results.

4 Cooperative solution

In the previous section we determined the non-cooperative (*status-quo*) strategies for both forest owners and non-owners. We saw that forest owners find it optimal to deforest as much as possible and to not afforest. On the other hand, non-owners suffer a negative environmental externality coming from the depletion of the forest via the *reduced-carbon-sequestration effect.* A relevant question to address is whether cooperation can improve welfare. The collectively optimal solution can be obtained by jointly optimizing the payoff functionals of the two players, that is,

$$\max_{\substack{0\leq A(t)\leq A_{\max},\\ 0\leq D(t)\leq D_{\max},\\ V_{\min}\leq V(t)\leq V_{\max}}} \int_0^T e^{-rt}\left[R(F(t),D(t))+G(E(t))-L(S(t))\right]dt-\phi(S(T))e^{-rT}$$

$$\begin{aligned}\text{s.t.:}\quad & \dot{F}(t)=A(t)+\eta F(t)-D(t), \quad \bar{F}\geq F(t)\geq 0,\ F(0)=F_0,\\ & \dot{E}(t)=V(t)E(t), \quad E(t)\geq 0,\ E(0)=E_0,\\ & \dot{S}(t)=E(t)-\varphi F(t)-W, \quad S(t)\geq 0,\ S(0)=S_0,\end{aligned}$$

where A, D and V are the three control variables. The joint payoff is maximized subject to the dynamics of the forest area, emissions, and stock of greenhouse gases in the atmosphere.

The Hamiltonian of the joint-optimization problem is

$$\begin{aligned}H^c\left(F,E,S,A,D,V,\lambda_F^c,\lambda_S^c,\lambda_E^c\right) &= R(F,D)+G(E)-L(S)+\lambda_F^c[A+\eta F-D]\\ &\quad+\lambda_S^c[E-\varphi F-W]+\lambda_E^c VE,\end{aligned}$$

where λ_F^c, λ_E^c, λ_S^c denote the co-state variables associated with the forest stock, emissions and the stock of GHGs, respectively. All the variables with a superscript c refer to cooperation as opposed to the non-cooperative outcomes computed before.

The Lagrangian of the cooperative problem is given by

$$\begin{aligned}\mathcal{L}^c\left(F,E,S,A,D,V,\lambda_F^c,\lambda_S^c,\lambda_E^c,w_1^c,w_2^c\right) &= H^c\left(F,E,S,A,D,V,\lambda_F^c,\lambda_S^c,\lambda_E^c\right)\\ &\quad+w_1^c D+w_2^c\left(D_{\max}-D\right),\end{aligned}$$

where w_1^c and w_2^c are the Lagrange multipliers associated with the deforestation rate.

The first-order optimality conditions are the following:

- State dynamics:

$$\begin{aligned}
\dot{F} &= A + \eta F - D, \quad \bar{F} \geq F \geq 0, \quad F(0) = F_0, \\
\dot{E} &= VE, \quad E \geq 0, \quad E(0) = E_0, \\
\dot{S} &= E - \varphi F - W, \quad S \geq 0, \quad S(0) = S_0,
\end{aligned} \tag{18}$$

- Adjoint equations:

$$\begin{aligned}
\dot{\lambda}_F^c &= (r - \eta)\lambda_F^c + 2\theta y^2 \gamma\delta(D + \gamma\delta F) + 2p_A\beta\frac{F}{\bar{F}} \\
&\quad + p_A(\bar{x} - 2\beta) - \bar{p}y\gamma\delta + \varphi\lambda_S^c, \quad \lambda_F^c(T) = 0,
\end{aligned}$$

$$\dot{\lambda}_S^c = r\lambda_S^c + 2c(S - \underline{S}), \quad \lambda_S^c(T) = 2c\phi[\underline{S} - S(T)], \tag{19}$$

$$\dot{\lambda}_E^c = (r - V)\lambda_E^c - a + bE - \lambda_S^c, \quad \lambda_E^c(T) = 0, \tag{20}$$

- Optimality conditions for D :

$$\begin{aligned}
&\frac{\partial \mathcal{L}^c}{\partial D} = -2\theta y^2 \left[\gamma\delta F + D\right] + \bar{p}y + p_A\psi\bar{Z} + w_1 - w_2 - \kappa_2 - \lambda_F^c = 0. \\
&w_1^c \geq 0, \;\; w_1^c D = 0, \;\; D \geq 0, \\
&w_2^c \geq 0, \;\; w_2^c(D_{\max} - D) = 0, \;\; D_{\max} \geq D,
\end{aligned}$$

- Optimality conditions for A :

$$A(t) = \begin{cases} 0 & \text{if } \frac{\partial \mathcal{L}^c}{\partial A} = -\kappa_1 + \lambda_F^c(t) < 0, \\ \widetilde{A} \in [0, A_{\max}] & \text{if } \frac{\partial \mathcal{L}^c}{\partial A} = -\kappa_1 + \lambda_F^c(t) = 0, \\ A_{\max} & \text{if } \frac{\partial \mathcal{L}^c}{\partial A} = -\kappa_1 + \lambda_F^c(t) > 0. \end{cases}$$

- Optimality conditions for V :

$$V(t) = \begin{cases} V_{\min} & \text{if } \frac{\partial \mathcal{L}^c}{\partial V} = \lambda_E^c(t)E(t) < 0, \\ \widetilde{V} \in [V_{\min}, V_{\max}] & \text{if } \frac{\partial \mathcal{L}^c}{\partial V} = \lambda_E^c(t)E(t) = 0, \\ V_{\max} & \text{if } \frac{\partial \mathcal{L}^c}{\partial V} = \lambda_E^c(t)E(t) > 0. \end{cases}$$

Note that the optimal afforestation rate and speed of adjustment of emissions are bang-bang policies because the Lagrangian is linear in these variables. Further, although λ_F^c appears, as in the non-cooperative case, in the optimality conditions for A and D, there is an important difference, that is, λ_F^c now captures the negative valuation of an extra hectare of forest (forest owners) and the positive effect that increasing forest area has on carbon sequestration (non-owners). Therefore, λ_F^c can take either positive or negative values depending on which effect dominates. Furthermore, unlike in the non-cooperative case where the sign of λ_F^c was constant along the planning horizon for both players, now nothing prevents this sign from changing over time. Hence, it is possible that we have a switch in either the afforestation rate or the deforestation rate or in both throughout the planning horizon.

To solve for λ_F^c, we need the analytical expression for F, which depends on both A and D (see (18)). In the non-cooperative scenario it was possible to analytically characterize the solution to the forest owners' problem by supposing *ex-ante* that we were in the right case of figure, and then verifying, *ex-post*, that our first-order conditions were indeed satisfied (see Appendix B for more details). This reasoning was possible because the optimal afforestation and deforestation rates were constant. In the present case however, we can have a policy switch on A and/or D at any time. Therefore, the value of λ_F^c depends on the switching time on A and D. The implication is that the first-order conditions will be satisfied for all $t \in [0, T]$ only if the exact switching time for both variables is chosen.

From (20) we see that λ_E^c depends on λ_S^c, and from (19) we have that S is a function of F. Therefore, to obtain λ_E^c we need to know the evolution of the forest stock, which depends on the applied policies for afforestation and deforestation. As it turns out, not only do we have a potential switch of regimes for all three controls, but the switches themselves are interdependent.

One can obtain the analytical expressions for the evolution of the state and co-state variables for all possible cases (i.e., before and after the switch). But just as it happened with the problem of the non-forest-owner group, it is not possible to derive the exact switching times analytically.

Denote now by t_A^c, t_D^c and t_V^c the switching time for A, D and V, respectively. We evaluated the discounted intertemporal sum of joint payoffs for all possible combinations of integer switching times (t_A^c, t_D^c, t_V^c) using a similar algorithm as before. See Table 2 for a sketch of the algorithm.

Table 2: Sketch of the algorithm used to obtain $\tilde{t}_A^c$, $\tilde{t}_D^c$, $\tilde{t}_V^c$

Fix the joint intertemporal discount rate r
for all integer planning horizons $T \in [1, ..., 100]$
  for all possible integer switching times t_A^c
    for all possible integer switching times t_D^c
      for all possible integer switching times t_V^c **do**

$$\begin{aligned}\text{JointPayoff}(t_A^c, t_D^c, t_V^c, T) &= \text{Discounted sum of revenues of FO}\\ &+ \text{Discounted sum of payoffs of NO}\\ &+ \text{Scrap-value function of NO}\end{aligned}$$

      end
    end
  end
end
Select the t_A^c, t_D^c, t_V^c whose JointPayoff is greater for each value of T.

The only difference from the previous algorithm is that now the computational complexity is increased as a consequence of the multiplicity of cases. Denote by $\tilde{t}_A^c$, $\tilde{t}_D^c$, $\tilde{t}_V^c$ the three integer switching times that yield greater intertemporal payoffs. We computed $\tilde{t}_A^c$, $\tilde{t}_D^c$, $\tilde{t}_V^c$ for $T \in \{1, 2, \ldots, 100\}$ and for $r \in \{0.01, 0.02, 0.03\}$. Again, the results are linked to the length of the planning horizon and the discount rate used. We observe that the solutions obtained can be classified into four different groups that coincide with four regions of the parameter space. We denote them by Z_1 to Z_4. The boundaries of regions $Z_1 - Z_4$ are related to parameter T. We denote the limits to these regions by T_1, T_2 and T_3. Figure 5 is a schematic representation of the solution.

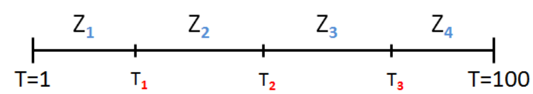

Figure 5: Cooperation timeline is a function of T

The results, which are summarized in Table 3, call for the following comments:

(i) If the planning horizon is short (i.e., $T < T_1$) we are in region Z_1 and the cooperative solution coincides with the non-cooperative one (i.e., the cooperative solution brings no gain). The label not applicable ($N.A.$) is used here to denote that there is no switching time and that the solution coincides with the *status quo*.

(ii) If we are in region Z_2 (i.e., $T_1 \leq T < T_2$) then it is jointly optimal to afforest at the maximum rate for some time and then to switch to afforestation $A_{\min}$ at some time before the end of the planning horizon. It is not optimal to afforest all the time and we have that $A^c = A_{\max}$ if $t < \tilde{t}_A^c$ and $A^c = A_{\min}$ if $t \geq \tilde{t}_A^c$. We use the notation $\tilde{t}_A^c = f(T)$ to denote the fact that the switching time depends on T. In fact $f(T)$ is an increasing function of T. Clearly, for larger values of T, it is optimal to switch later. The same reasoning applies for $\tilde{t}_D^c$. In this case, though, we have that $D^* = D_{\min}$ if $t < \tilde{t}_D^c$ and $D^* = D_{\max}$ if $t \geq \tilde{t}_D^c$.

(iii) If we are in region Z_3 (i.e., $T_2 \leq T < T_3$), then it is optimal to apply $A^* = A_{\max}$ and $D^* = D_{\min}$ all along. We have used the notation $\tilde{t}_A^c = \tilde{t}_D = T$ to differentiate it from label $N.A.$ Recall that label $N.A.$ was used to denote that there is no switch and that the optimal policy is identical to the *status quo* one (i.e., $A^c = A_{\min}$ and $D^c = D_{\max}$ $\forall t \in [0, T]$) whereas in region Z_3, we have that there is no switch either, but the optimal policy is to apply $A^c = A_{\max}$ and $D^c = D_{\min}$ throughout.

(iv) Finally, region Z_4 is identical to region Z_3 except for the emissions policy. If $T \geq T_3$ then it is certain that we will have a jump from $V_{\max}$ to $V_{\min}$ at some point in time $\tilde{t}_V^c$. The time of the switch is also a function of T.

The impact of cooperation is more intense and the solution is more environmentally friendly as we move from region Z_1 (no gain from cooperation) to region Z_4. When the discount rate is smaller, the environmental damage is further internalized. Table 4 shows the values of T_1 to T_3 for different values in

Table 3: Jointly optimal policies are a function of T

Switch	Z_1	Z_2	Z_3	Z_4
$\tilde{t}_A$	$N.A.$	$\tilde{t}_A = f(T)$	T	T
$\tilde{t}_D$	$N.A.$	$\tilde{t}_D = g(T)$	T	T
$\tilde{t}_V$	$N.A.$	$N.A.$	$N.A.$	$\tilde{t}_V = h(T)$

the discount rates. It is not surprising that when the discount rates are smaller, the threshold planning horizons (T_1, T_2, T_3) between regions Z_1, Z_2, Z_3 and Z_4 are shifted downwards (See Table 4).

Table 4: Threshold times are a function of the discount rate

Discount	T_1	T_2	T_3
$r = 1\%$	11	19	36
$r = 2\%$	12	20	38
$r = 3\%$	12	21	41

4.1 Robustness analysis

Most of the parameters used in the model proposed for forest owners were obtained from the FAO's Forest Resources Assessment (2006) or, when unavailable from the FAO, from other sources in the literature (see Appendix A). Parameters a and b used in the the non-forest-owner group's payoff function have been calibrated to fit the worldwide GDP while making sure that the emission gains are always increasing and concave for a relevant time frame. Finally, parameter c captures the environmental damage coming from the accumulation of greenhouse gases in the atmosphere and is key to the model. There is great uncertainty regarding the exact impact of emissions on climate change and, therefore, on damage. For this reason, attempting to estimate parameter c is a hard task. In an effort to account for part of this uncertainty, we perform a sensitivity analysis and consider two cases: First in case 1, we suppose the environmental damage parameter (parameter c) to be one-third greater than the benchmark case used so far. Then in case 2, parameter c is supposed to be one-third below the same benchmark.

We recomputed the cooperative outcomes obtained in the previous section for these two cases. We do not observe any qualitative changes. Just as before, we have four areas of interest, $Z_1 - Z_4$. The behaviour in these four areas is exactly the same. The only difference that we observe is that cooperation will be more easily (i.e. earlier) achieved when the environmental damage is higher (case 1). The results are summarized in Table 5 below.

Table 5 shows the threshold times ($T_1 - T_3$) for cases 1 and 2. These thresholds are shifted downwards when c increases (case 1) and upwards when c decreases (case 2). A downward shift in T_1 indicates that the minimum planning

Table 5: Robustness of solution to changes in environmental damage

Discount	Benchmark			Case 1			Case 2		
	T_1	T_2	T_3	T_1	T_2	T_3	T_1	T_2	T_3
$r = 1\%$	11	19	36	8	15	31	15	25	42
$r = 2\%$	12	20	38	9	16	33	17	27	46
$r = 3\%$	12	21	41	9	17	36	18	29	49

horizon, beyond which cooperation brings gains, is reduced. Analogously, if T_2 and T_3 are reduced this will mean that it is optimal to enhance cooperation for shorter planning horizons than before, and vice versa for upwards shifts of the thresholds.

To sum up, our results seem quite robust to changes in parameter c and even if the thresholds are affected, the structure of the solutions does not change and cooperation is still strictly welfare improving for all the scenarios studied, regardless of the value of c used.

5 Sharing the gain of cooperation

In this section, we determine a time-consistent allocation of the dividend of cooperation among the two players. We have shown that joint payoffs are greater in the cooperative setting provided that $T \geq T_1$. This is due to the damage reduction generated by increased afforestation effort and lower deforestation rates. Cooperation, however, does not bring gains to both players. The non-forest-owner group gains from the lower environmental damage, while forest owners lose by applying forest policies that are environmentally friendly but revenue harming.

Let (x_τ, τ) be the position of the game at time $\tau \in [0, T]$ and a state-vector value x_τ. Denote by $J_i^c(x_0, t_0)$ the payoff-to-go that player $i \in \{\mathcal{O}, \ominus\}$ obtains if the game is played cooperatively throughout the planning horizon, and by $J_i^{nc}(x_0, t_0)$ its non-cooperative counterpart. The difference $J_\ominus^c(x_0, t_0) - J_\ominus^{nc}(x_0, t_0)$ measures the individual gain that non-owners obtain from cooperation. By the same token $J_\mathcal{O}^{nc}(x_0, t_0) - J_\mathcal{O}^c(x_0, t_0)$ represents the loss that forest owners have in the cooperative setting *vis-à-vis* the non-cooperative one. These two quantities are a function of T and r. We compare the cooperative gains of non-owners and the cooperative losses of forest owners in Figure 6 for $r = 2\%$.

The cooperative gain by the non-forest-owner group is represented by the solid line, and the loss by forest owners by the dashed one. The vertical difference between these two lines measures the dividend of cooperation given by

$$DC = \left[J_\ominus^c(x_0, t_0) + J_\mathcal{O}^c(x_0, t_0)\right] - \left[J_\ominus^{nc}(x_0, t_0) + J_\mathcal{O}^{nc}(x_0, t_0)\right],$$

for any given planning horizon T. We obtain empirically that $DC > 0$ for $T \geq T_1$ (with $T_1 = 11$ years), and $DC = 0$, otherwise. This means that, unless the planning horizon is longer than T_1 (which is 11 years for $r = 2\%$), cooperation is

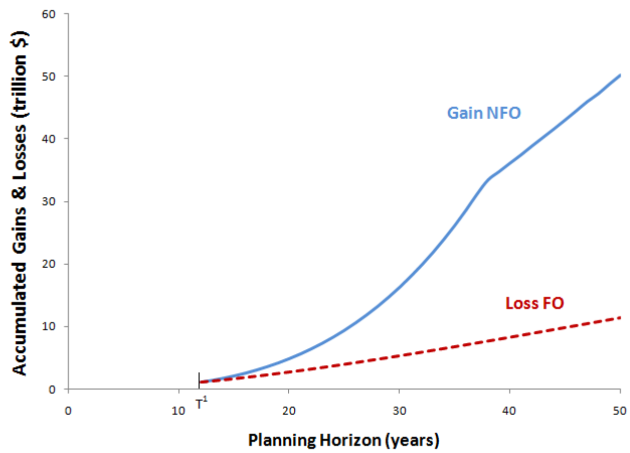

Figure 6: Cooperation gains and losses by NFO and FO

useless. For intermediate values of T, i.e., $T_1 \leq T < T_3$, we have $DC > 0$, and it is optimal to mitigate future damages by increasing afforestation and decreasing deforestation, but not to abate emissions. If the planning horizon is sufficiently long, that is, $T \geq T_3$, then it is optimal both to mitigate (from the beginning) and to abate emissions (from time $\tilde{t}_V^c$ onwards). As emissions abatement has a greater cost than increasing afforestation or decreasing deforestation, it is preferable to start by applying less costly measures first and then move into more costly ones as environmental damages increase.

Although the total dividend of cooperation is by virtue of joint optimization always nonnegative, this does not mean that cooperation is Pareto improving. In our case, it is clear that for cooperation to be implemented, the non-owners need to compensate forest owners for their losses. There are many solution concepts in cooperative games that address the problem of sharing DC. We adopt here the often used Nash-bargaining procedure, which gives a unique, fair and Pareto-improving solution.[12] The Nash-bargaining solution (NBS) allocates to each player his non-cooperative outcome plus half of the dividend of cooperation, that is,

$$J_i^{NBS}(x_0, t_0) = J_i^{nc}(x_0, t_0) + \frac{1}{2} \sum_{i \in \{\mathcal{O}, \ominus\}} \left(J_i^c(x_0, t_0) - J_i^{nc}(x_0, t_0) \right). \qquad (21)$$

5.1 Time-consistent sharing schedule

Although the Nash-bargaining outcomes defined in (21) are Pareto-improving with respect to non-cooperative outcomes, it does not guarantee that the play-

[12]The unfamiliar reader with the Nash-bargaining solution can consult, e.g., Wikipedia (http://en.wikipedia.org/wiki/Bargaining_problem) for a quick introduction.

ers will indeed continue to implement over time their part of the cooperative solution. In fact, the agreement will not be sustained if it is optimal, for at least one of them, to deviate to a non-cooperative mode of play at an intermediate date $\tau \in (0, T]$. This would mean that the agreement designed at initial date for the whole duration of the game is not time consistent. (For a tutorial on time consistency in differential games, the reader may consult Yeung and Petrosjan (2005) or Zaccour (2008).) Formally, we say that a cooperative solution (here NBS) is time consistent at (x_0, t_0) if, at any position (x_τ^*, τ), and for all $\tau \in [t_0, T]$, it holds that

$$J_i^{NBS}(x_\tau^*, \tau) \geq J_i^{nc}(x_\tau^*, \tau), \qquad i \in \{\mathcal{O}, \ominus\}, \tag{22}$$

where x^* denotes the cooperative state trajectory. Note that the comparison of payoffs-to-go in (22) at any $\tau \in [t_0, T]$ is carried out along the cooperative state trajectory, that is, under the assumption that the players have cooperated until τ.

Solving the time-consistency problem amounts to finding payment functions $\omega_i(t), i \in \{\mathcal{O}, \ominus\}, t \in [t_0, T]$, such that the following two properties hold:

$$\text{Full allocation} \quad : \quad \int_{t_0}^{T} e^{-rt}\omega_i(t)dt = J_i^{NBS}(x_0, t_0), \tag{23}$$

$$\text{Time consistency} \quad : \quad J_i^{NBS}(x_0, t_0) = \int_{t_0}^{\tau} e^{-rt}\omega_i(t)dt + e^{-r\tau}J_i^{NBS}(x_\tau^*, \tau). \tag{24}$$

The first property states that the total payments that each player recieves overtime must correspond to what he is entitled to, as determined by the Nash-bargaining solution. To interpret the second condition, assume that the players wish to renegotiate the initial agreement at (any) intermediate instant of time τ. At this moment, the position of the game is (x_τ^*, τ), meaning that cooperation has prevailed from the initial time until τ, and that each player i would have been allocated a stream of monetary amounts given by the first right-hand-side term. Now, if the subgame starting with initial condition $x(\tau) = x_\tau^*$, is played cooperatively, then player i will get his NBS-value component in this game given by the second right-hand-side term of (24). If what he/she has been allocated until τ and what he will be allocated from this date onward add up to his payoff in the original agreement, i.e., his NBS value $J_i^{NBS}(x_0, t_0)$, then a renegotiation would leave the original agreement unaltered. If one can find a vector $\omega(t) = (\omega_{\mathcal{O}}(t), \omega\ominus(t))$ such that (24) holds true, then the allocation over time $\omega(t)$ is time consistent. To obtain the value $\omega_i(t), t \in [t_0, T]$, it suffices to differentiate (24), that is,

$$\omega_i(\tau) = rJ_i^{NBS}(x_\tau^*, \tau) - \frac{d}{d\tau}\left(J_i^{NBS}(x_\tau^*, \tau)\right). \tag{25}$$

The above formula has a nice interpretation. It allocates to player i at time τ the interests on cooperative payoff-to-go, minus the variation over time of this payoff-to-go.

We have computed forest owners' NBS payoffs-to-go $(J_{\mathcal{O}}^{NBS}(x_\tau^*, \tau))$ and their non-cooperative payoffs-to-go $(J_{\mathcal{O}}^{nc}(x_\tau^*, \tau))$. The results are plotted in Figure 7.

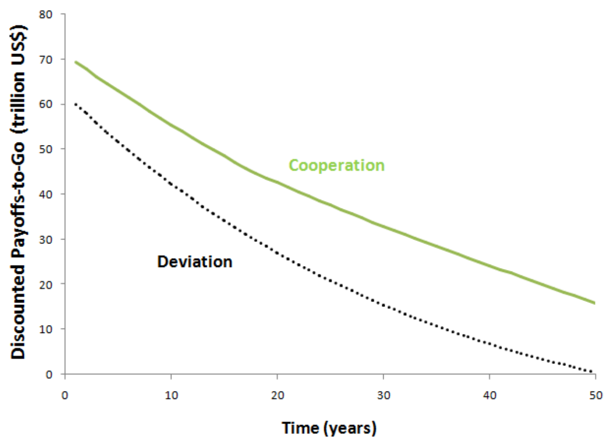

Figure 7: Time Consistency: Non-forest-owner group

It is straightforward to see that NBS outcomes dominate their non-cooperative counterparts for any time τ. Figure 8 shows the same result for non-forests owners. The economic interpretation is that the compensating mechanism based on the Nash-bargaining solution is time consistent for both players, that is, cooperation is implementable and sustainable overtime.

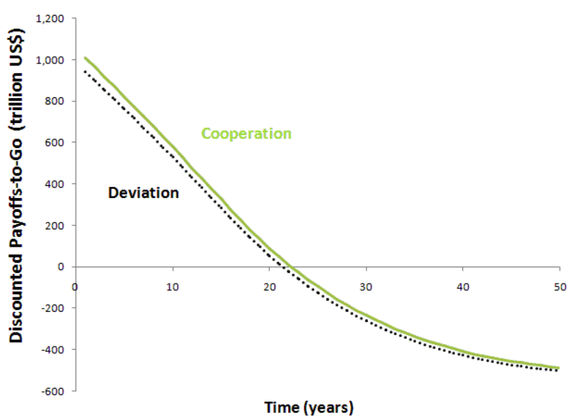

Figure 8: Time Consistency Non-forest Owners

6 Conclusions

Forests play an important role in mitigating climate change. In this paper we propose a model with two players: forest owners and the non-forest-owner group. Forest owners have an incentive to deforest to increase their economic revenues, while non-owners suffer a negative externality from deforestation due to the so-called *reduced-carbon-sequestration effect,* which states that a tree that is cut cannot grow and hence cannot sequester carbon. We model the economic incentives of both types of players and explore the conditions that make environmental cooperation strictly welfare improving. Cooperation brings greener outcomes and makes it possible to partly internalize the positive externality created by carbon sequestration by forests.

Three different mechanisms to reduce GHG accumulation are proposed: abatement of emissions, increases in afforestation, and decreases in deforestation. The results show that when the perceived environmental damage is small (i.e., short planning horizons and/or high intertemporal discount rates), cooperation brings little or no gain. However, as the environmental damages increase, it becomes jointly optimal to have some afforestation effort and deforestation reduction. If the environmental damages coming from the excessive accumulation of greenhouse gases are sufficiently high, then it will be optimal to combine forestation efforts (reduced deforestation and increased afforestation) with emissions abatement. Reducing emissions is more expensive but also more effective in mitigating future environmental damage coming from the excessive accumulation of GHGs.

Our results convey a doubly positive message: First, considering forests' carbon-sequestration potential can make a significant difference toward stopping the destruction of the forests. Second, international cooperation can bring sound economic and environmental gains. Cooperation however will not arise spontaneously. For cooperation to exist, some sort of intertemporal compensating transfer mechanism is needed. It is important to design this intertemporal transfer mechanism correctly; otherwise the agents may have an economic incentive to withdraw from it, which in turn, will lead to worse environmental outcomes. In order for an environmental agreement to be credible, time consistency is required, i.e., it is economically optimal for all the players involved to comply with the agreement at all times. We show that applying an intertemporal decomposition of the Nash bargaining scheme allows us to obtain time consistent outcomes.

The results obtained are very promising and can be applied to designing time-consistent intertemporal payments in REDD and REDD+ agreements. That being said, there are many aspects that were not considered and that call for a critical interpretation of the results: carbon sequestration by the oceans may be affected by the excessive seawater acidification. Also a more comprehensive dynamics of the accumulation of greenhouse gases should consider emissions related to land-use change. More thorough research should integrate these aspects.

Appendix A: Description of variables and parameters

State variables

F: **Stock of forest**

Forest surface area of the world measured in hectares. The current stock of forest F_0 was estimated by the FAO (2006) at 3952 million hectares in 2005. Parameter $\bar{F} = F_{\max}$ is estimated, for 1750 C.E., to have been 42% of the globe's surface[13] (i.e., 13067 million hectares excluding Antarctica and Greenland). This gives us a value for $F_{\max}$ of approximately 5500 million hectares. Consequently we require that $F(t) \in [F_{\min}, F_{\max}] = \left[0 \cdot 10^9,\ 5.5 \cdot 10^9\right]$.

E: **Yearly emissions of CO_2**

These are world yearly emissions of CO_2 measured in metric tons. In 2005, total CO_2 emissions (E_0) amounted to 28.2 billion metric tons, i.e., 7.7 GtC (gigatons of carbon).[14]

S: **Cumulated quantity of CO_2 in the atmosphere**

The cumulated stock of CO_2 in the atmosphere is measured in gigatons. The current stock of CO_2, which we denote by S_0 has been estimated to be approximately 3000 Gt, which is equivalent to 800 GtC (383-387 ppmv) in 2007.[15]

Control variables

A : **Yearly afforestation** $[A_{\min}, A_{\max}] = [0,\ 3 \cdot 10^6]$

For the period 1990-2005, the FAO (2006) estimates the world yearly afforestation rate at 2.8 million hectares.

D : **Yearly deforestation** $[D_{\min}, D_{\max}] = [0,\ 13 \cdot 10^6]$

For the same period, the FAO (2006) estimates the average global deforestation rate at 13 million hectares per year.

V : **CO_2 emissions adjustment rate** $[V_{\min}, V_{\max}] = [-0.015,\ 0.03]$

During the decade from 1990 to 2000, world CO_2 emissions increased roughly 1% every year.[16] Only a few countries (e.g., Germany, the United Kingdom, Denmark, Finland, some Eastern European countries and former Soviet Republics) were able to reduce their emissions. Germany was the most outstanding case, achieving a 1.8% yearly cumulative decrease. On the other hand China's emissions increased at a rate of 3% per annum. All the other big economies lie

[13] Source: http://www.geo.vu.nl/~renh/deforest.htm
[14] Source: EIA (2008).
[15] Source: NOAA (2007).
[16] Source: Bernstein et al. (2006) and EIA (2008).

somewhere in between. From 2000 to 2008, however, world emissions increased at a faster rate, 3.4% yr^{-1}, (Le Quéré et al., 2009) with a probable decrease during the next two years (2009-2010) due to the world crisis. Following these observations, we set both the lower and upper bounds on V. As a benchmark scenario, we chose $V \in [V_{\min}, V_{\max}] = [-0.015, 0.03]$.

Parameters

a, b: Emissions-output ratio parameters

Parameters a and b are chosen to ensure that (i) $G(E)$ in (8) is increasing and concave throughout, and (ii) $G(E_0)$ equals the world's GDP, as estimated by the World Bank for the year 2008.[17] $a = 2100$, $b = 4 \cdot 10^{-9}$.

c: Environmental damage parameter

This parameter captures the impact of greater GHG concentration levels on the welfare of individuals. $c = 1.5 \cdot 10^{-11}$.

$\underline{S}$: Pre-industrial CO_2 concentration level

Parameter $\underline{S}$ was set to match preindustrial levels, i.e., 284 ppmv in year 1832,[18] which is equivalent to 587 GtC.

κ_1: Per-hectare afforestation cost

The World Bank estimates the cost for seedling at roughly 40 US$ per thousand seedlings. Considering that the number of seedlings per hectare is equal to approximately 2000, this amounts to approximately $80 per hectare of forest. Afforestation costs also include other costs (e.g., labour) that fluctuate by country. The World Agroforestry Centre and other NGO organisations provide estimates that range from $180/ha to around $1000/ha.[19] We have chosen the round and representative value of $500/ha.

κ_2: Per-hectare deforestation cost

The Bureau of Business and Economic Research of Montana University estimates the costs of ground-based logging per green ton of harvest for the year 2006 at $22.70.[20] A green ton is equivalent to 907 kg (2000 pounds of undried biomass material). The density of wood is typically 500 kg/m^3. For a representative Douglas-fir plantation (530 kg/m^3) we obtain a deforestation cost of $13.26/$m^3$. If the yield per hectare is equal to 110 m^3/ha (see the estimation of y below), then we obtain an estimate of the deforestation cost per hectare of $1459/ha.

[17] http://web.worldbank.org/
[18] Source: NOAA.
[19] See e.g., www.villageprojectsint.org and www.edenprojects.org
[20] www.bber.umt.edu/pubs/forest/prices/loggingCostPoster.pdf

η: **Natural growth rate of the forest**

The FAO (2006) estimates the average yearly natural expansion of the world's forests to be equal to 2.9 million hectares, i.e., $\eta\ F = 2.9 \cdot 10^6$ ha. Parameter η is dimensionless and can be estimated accordingly: $\eta = 7.34 \cdot 10^{-4}$.

φ: **Carbon absorption rate**

This parameter is measured in metric tons of CO_2 equivalent absorbed per hectare of forest and year. According to Le Quéré et al. (2009), during the decade from 1990 to 2000 forests absorbed 2.6 PgC yr^{-1}(i.e., 2.6 GtC), which amounts to 9.53 $GtCO_2$. The world's total forest area equals 3952 million hectares. If we consider a homogeneous forest, its mean yearly carbon sequestration is 2.412 tonnes of CO_2 $ha^{-1}yr^{-1}$.

W: **Carbon absorption rate by oceans**

Le Quéré et al. (2009) estimate that oceans were able to sink, on average, 2.2 PgC yr^{-1}(8.07 $GtCO_2$ yr^{-1}) during the period 1990-2000. We have set parameter W equal to their estimate.

y: **Per-hectare timber yield**

Timber yield is measured in m^3 of wood per hectare. According to the FAO (2006), the mean wood content of a hectare of forest land in 2005 is equal to 110 m^3.

β: **Lower productivity due to forest depletion**

Eswaran et al. (2001) estimate the productivity loss as a consequence of land degradation, erosion, and desertification for the African continent at 8.2% of the average productivity. Average land productivity is measured by $\bar{x}$ (see the estimation below). Parameter β is thus equal to 0.061 (8.2% of $\bar{x}$).

γ: **Selective logging yield, fraction of average yield**

The selective logging yield is measured as a fraction of the average yield. When forests are managed for wood production, they produce as much as 1-3 m^3 per hectare (in other words $y\gamma$ =1-3 m^3). Following Andrés-Domenech et al. (2011), we set the value of $\gamma = 1.5\%$.

δ: **Fraction of forests selectively logged**

Share of the world's forests selectively logged. Following the FAO (2006), parameter δ has been calibrated at 30% to fit the world's current yearly production of wood.

θ: **Slope of wood demand**

According to the FAO (2006), the commercial value of all wood (i.e., roundwood and fuelwood) in 2005 was US$64 billion per year of which only 7 billion corresponded to fuelwood. Current world production equals 3400 million m^3. The average price for both types of wood is $18.8/$m^3$. The FAO (1997) gives the elasticity of demand for several countries and several types of wood.[21] A representative value of both the mean and median price elasticity of wood is -0.50. We have approximated an iso-elastic curve by a linear one in an interval of 2000 million m^3 centred at 3400 million m^3 such that the average elasticity inside the interval equals -0.50. The slope of our demand can then be computed and we obtain $\theta = 2.7 \cdot 10^{-9}$.

$\bar{p}$: **Choke price of wood**

With the average price of wood and the slope of demand computed above, we can retrieve the choke price of our inverse demand function and obtain $\bar{p} = 27.98$ (US$ per m^3).

ψ: **Extra productivity of deforested land**

Parameter ψ denotes the productivity gain of land after deforestation. It is measured as a fraction of average productivity. We adopt $\psi = 0.3$ following Andrés-Domenech et al. (2011).

p_A: **Average price of representative agricultural product**

Measured in US$ per metric ton. To determine the average price of the representative agricultural goods we took four representative commodities (i.e., cocoa, coffee, cotton and sugar) from the FAO (2004). These four commodities are related to deforestation processes. The net economic yield per hectare of crop ranges from $1660/ha for coffee to $771/ha for cocoa. The mean yield equals $1141/ha. Cotton is the more representative of the four in terms of prices and economic yield ($1467 per metric ton and $1088/ha). We use the price of cotton as a reference.

$\bar{x}$: **Average land productivity**

Measured in tons per hectare. Average land productivity was computed with the same four crops used to obtain p_A. Parameter $\bar{x}$ is set equal to 0.742 metric tons per hectare.

[21] Most elasticity values are found between -0.25 and -0.75

Appendix B: Proof of Proposition 1

The Hamiltonian of the forest owners' control problem is[22]

$$\begin{aligned} H^{\mathcal{O}}(F,A,D,\lambda) &= [\bar{p}-\theta(yD+y\gamma\delta F)]\,y(D+\gamma\delta F)\\ &+ p_A\left[\bar{x}+\frac{\psi\bar{x}}{\bar{F}-F}D-\beta\frac{\bar{F}-F}{\bar{F}}\right]\left(\bar{F}-F\right)-\kappa_1A-\kappa_2D\\ &+ \lambda\,[A+\eta F-D]\,, \end{aligned}$$

where λ denotes the co-state variable associated with the forest stock. The Lagrangian of forest owners can be written as

$$\mathcal{L}^{\mathcal{O}}(F,A,D,\lambda,w_1,w_2)=H^{\mathcal{O}}(F,A,D,\lambda)+w_1D+w_2\,(D_{\max}-D)\,,$$

where $w_1(t)$ and $w_2(t)$ are the Lagrange multipliers associated with the non-negativity condition $D(t)\geq 0$ and $D(t)\leq D_{\max}$.[23]

The first-order optimality conditions read

$$\max_{\substack{0\leq A\leq A_{\max}\\ 0\leq D\leq D_{\max}}} \mathcal{L}^{\mathcal{O}}(F,A,D,\lambda,w_1,w_2)\,, \tag{26}$$

$$\begin{aligned} \dot{F} &= A+\eta F-D, \quad \bar{F}\geq F\geq 0,\; F(0)=F_0, &(27)\\ \dot{\lambda} &= r\lambda-\frac{\partial\mathcal{L}^{\mathcal{O}}}{\partial F}, \quad \lambda(T)=0, &(28)\\ w_1 &\geq 0,\; w_1D=0,\; D\geq 0,\\ w_2 &\geq 0,\; w_2(D_{\max}-D)=0,\; D_{\max}\geq D. \end{aligned}$$

The necessary condition for the maximization problem in (26) with respect to the deforestation rate reads

$$\frac{\partial\mathcal{L}^{\mathcal{O}}}{\partial D}=0;\quad -2\theta y^2\,[\gamma\delta F+D]+\bar{p}y+p_A\psi\bar{x}+w_1-w_2-\kappa_2-\lambda=0. \tag{29}$$

With respect to the afforestation rate, A, we have a Lagrangian that is linear in A and $\frac{\partial\mathcal{L}^{\mathcal{O}}}{\partial A}=-\kappa_1+\lambda$. The optimal afforestation rate is a bang-bang policy as follows:

$$A^{nc}(t)=\begin{cases} 0 & \text{if } -\kappa_1+\lambda(t)<0,\\ \widetilde{A}\in[0,A_{\max}] & \text{if } -\kappa_1+\lambda(t)=0,\\ A_{\max} & \text{if } -\kappa_1+\lambda(t)>0. \end{cases} \tag{30}$$

The differential equation (28) for the co-state variable reads

$$\begin{aligned} \dot{\lambda} &= (r-\eta)\lambda+2\theta y^2\gamma\delta(D+\gamma\delta F)+2p_A\beta\frac{F}{\bar{F}}+p_A(\bar{x}-2\beta)-\bar{p}y\gamma\delta,\\ \lambda(T) &= 0. \end{aligned} \tag{31}$$

[22]The time argument is eliminated when no confusion can arise.

[23]To simplify the notation, we do not include Lagrange multipliers associated with the non-negativity conditions on the other control variable, $A(t)$, because this variable enters the model in a linear way and the optimal afforestation policy is bang-bang.

It can be proved that a maximum deforestation rate ($D(t) = D_{\max}$ for all $t \in [0, T]$) and a minimum afforestation rate ($A(t) = 0$ for all $t \in [0, T]$) are the only ones to satisfy the optimality conditions established above.

Also, it can be shown that singular arcs are not possible. If such arcs were possible, $\lambda(t)$ would be equal to κ_1 during a non-trivial interval of time and along this interval, the following equation would be satisfied:

$$0 = (r-\eta)\kappa_1 + 2\theta y^2\gamma\delta(D+\gamma\delta F) + 2p_A\beta\frac{F}{\bar{F}} + p_A(\bar{x}-2\beta) - \bar{p}y\gamma\delta.$$

However, for our parameter domain $r-\eta > 0$ and $(2\theta y D_(t) - \bar{p})y\gamma\delta + p_A(\bar{x}-2\beta) > 0$ for all admissible values of $D(t)$. Therefore, the above equation cannot be satisfied, showing that singular arcs are not possible.

Replacing the optimal policies ($D(t) = D_{\max}$, $A(t) = 0$ for all $t \in [0, T]$) into the dynamics of the forest stock given in (27) we have

$$\dot{F} = \eta F - D_{\max}, \quad F(0) = F_0.$$

The solution to this differential equation is given by (14). Plugging (14) in equation (31) leads to

$$\begin{aligned} \dot{\lambda} &= (r-\eta)\lambda - \bar{p}y\gamma\delta + 2\theta y^2\gamma\delta D_{\max} + p_A(\bar{x}-2\beta) \\ &+ 2\left[\left(F_0 - \frac{D_{\max}}{\eta}\right)e^{\eta t} + \frac{D_{\max}}{\eta}\right]\left[\theta y^2\gamma^2\delta^2 + p_A\beta\frac{1}{\bar{F}}\right], \quad \lambda(T) = 0. \end{aligned}$$

From the integration of the above non-homogeneous linear differential equation, we get the following:

$$\begin{aligned} \lambda(t) &= \frac{1}{\eta - r}\left\{-\bar{p}y\gamma\delta + 2\theta y^2\gamma\delta D_{\max} + p_A(\bar{x}-2\beta)\right. \\ &+ \left. 2\left[\left(F_0 - \frac{D_{\max}}{\eta}\right)e^{\eta t} + \frac{D_{\max}}{\eta}\right]\left[\theta y^2\gamma^2\delta^2 + p_A\beta\frac{1}{\bar{F}}\right]\right\} + K_\lambda e^{(r-\eta)t}, \end{aligned}$$

where K_λ denotes the constant of integration.

This constant K_λ can be retrieved using the transversality condition for the co-state variable λ, $\lambda(T) = 0$. The final expression of the co-state optimal time path reads as in (15).

For our parameter domain, λ always takes negative values and increases over time to reach zero at T. Therefore, from (30), we conclude that the optimal afforestation policy is $A(t) = 0$ for all $t \in [0, T]$.

$$\frac{\partial \mathcal{L}^{\mathcal{O}}}{\partial D} = 0; \quad -2\theta y^2\left[\gamma\delta F + D\right] + \bar{p}y + p_A\psi\bar{x} + w_1 - w_2 - \kappa_2 - \lambda = 0.$$

Finally, to show that the optimal deforestation rate D^* is indeed $D_{\max}$ for all $t \in [0, T]$ (and hence $w_1 = 0$ and $w_2 \neq 0$), we replace the optimal time paths of F and λ given by (14) and (15), respectively, in equation (29). Given our parameters' values, we observe that if $w_2 = 0$, then the LHS of equation (29) is positive–instead of null–or all feasible F and D. The only way to avoid this contradiction is by having $w_2 \neq 0$. In other words, forest owners maximize their payoffs for $D = D_{\max}$ and the forest area along the optimal path decreases with time.

Appendix C: Proof of Proposition 2

The Hamiltonian of the optimal control problem of non-owners is

$$H^{\ominus}(F, S, E, V, \lambda_F, \lambda_S, \lambda_E) = aE - \frac{1}{2}bE^2 - c\left(S - \underline{S}\right)^2 + \lambda_F\left[A + \eta F - D\right] + \lambda_E VE + \lambda_S\left[E - \varphi F - W\right].$$

The first-order optimality conditions read[24]

$$\max_V H^{\ominus}, \tag{32}$$

$$\dot{F} = A + \eta F - D, \quad \bar{F} \geq F \geq 0,\ F(0) = F_0,$$

$$\dot{S} = E - \varphi F - W, \quad S \geq 0,\ S(0) = S_0, \tag{33}$$

$$\dot{E} = VE, \quad E \geq 0,\ E(0) = E_0,\ V \in [V_{\min}, V_{\max}], \tag{34}$$

$$\dot{\lambda}_F = r\lambda_F - \frac{\partial H^{\ominus}}{\partial F}, \quad \lambda_F(T) = 0, \tag{35}$$

$$\dot{\lambda}_S = r\lambda_S - \frac{\partial H^{\ominus}}{\partial S}, \quad \lambda_S(T) = -\frac{d\phi\left(S(T)\right)}{dS(T)}, \tag{36}$$

$$\dot{\lambda}_E = r\lambda_E - \frac{\partial H^{\ominus}}{\partial E}, \quad \lambda_E(T) = 0. \tag{37}$$

Since the Hamiltonian is linear in V, condition (32) is equal to $\frac{\partial H^{\ominus}}{\partial V} = \lambda_E E$ and leads to the following optimal bang-bang solution:

$$V^*(t) = \begin{cases} V_{\min} & \text{if} \quad \lambda_E(t)E(t) < 0, \\ \widetilde{V} \in [V_{\min}, V_{\max}] & \text{if} \quad \lambda_E(t)E(t) = 0, \\ V_{\max} & \text{if} \quad \lambda_E(t)E(t) > 0. \end{cases}$$

Equations (35), (36) and (37) can be written as

$$\dot{\lambda}_F = (r - \eta)\lambda_F + \varphi\lambda_S, \quad \lambda_F(T) = 0,$$

$$\dot{\lambda}_S = r\lambda_S + 2c(S - \underline{S}), \quad \lambda_S(T) = 2c\phi[\underline{S} - S\left(T\right)], \tag{38}$$

$$\dot{\lambda}_E = (r - V)\lambda_E - a + bE - \lambda_S, \quad \lambda_E(T) = 0, \tag{39}$$

where

$$\phi = \frac{1}{r}\left(1 - e^{-rT}\right).$$

[24] In order to simplify the presentation, we do not explicitly introduce the Lagrangian function and the restrictions on the state variables, but we check a posteriori that all these restrictions are satisfied. The time argument is also eliminated when no confusion can arise.

Let us assume $V(t) = V^*$ constant over the planning horizon, where V^* denotes either $V_{\min}, V_{\max}$ or $\widetilde{V}$. Solving the differential equation in (34), we can characterize the optimal trajectory of emissions, $E(t)$, which is given by (16).

The optimal path of the forest stock is known and given by equation (14), which is obtained from the forest owners' problem. Take equations (14) and (16), and plug them in (33). Integration of the resulting expression gives the expression in (17).

Given our parameter domain, it can be shown that both the optimal paths of emissions and stock of greenhouse gases are always greater than zero.

Using the expressions for the optimal paths of the state variables $F(t)$, $E(t)$ and $S(t)$ (expressions (14), (16) and (17) respectively), we can retrieve the optimal paths of the three co-state variables.

From the integration of the differential equation of the shadow price of the pollution stock, λ_S, in (38), we get

$$\begin{aligned}\lambda_S(t) =& K_S e^{rt} - \frac{2c}{r}\left[S_0 - \underline{S} - \frac{E_0}{V^*} + \frac{\varphi}{\eta}\left(F_0 - \frac{D_{\max}}{\eta}\right)\right] \\ &+ 2c\left[\frac{1}{r}\left(t + \frac{1}{r}\right)\left(W + \frac{\varphi}{\eta}D_{\max}\right) - \frac{\varphi}{\eta}\left(F_0 - \frac{D_{\max}}{\eta}\right)\frac{e^{\eta t}}{\eta - r} + \frac{E_0}{V^*}\frac{e^{V^* t}}{V^* - r}\right],\end{aligned}$$

where K_S denotes the integration constant, which can easily be determined using the transversality condition $\lambda_S(T) = 2c\phi[\underline{S} - S(T)]$. After replacing this constant on λ_S, the optimal path of the shadow price of the pollution stock reads

$$\lambda_S(t) = \Lambda_1 + \Lambda_2 e^{-r(T-t)} + \Lambda_3 t + \Lambda_4 e^{V^* t} + \Lambda_5 e^{\eta t},$$

where

$$\begin{aligned}\Lambda_1 &= -\frac{2c}{r}\left[S_0 - \underline{S} - \frac{E_0}{V^*} + \frac{\varphi}{\eta}\left(F_0 - \frac{D_{\max}}{\eta}\right) - \frac{1}{r}\left(W + \frac{\varphi}{\eta}D_{\max}\right)\right], \\ \Lambda_2 &= -\Lambda_1 - 2c\left[\left(\frac{W}{r} + \frac{\varphi}{\eta}\frac{D_{\max}}{r}\right)T - \frac{\varphi}{\eta}\left(F_0 - \frac{D_{\max}}{\eta}\right)\frac{1}{\eta - r}e^{\eta T} + \frac{E_0}{V^*}\frac{e^{V^* T}}{V^* - r}\right] \\ &\quad + 2c\phi[\underline{S} - S(T)], \\ \Lambda_3 &= \frac{2c}{r}\left(W + \frac{\varphi}{\eta}D_{\max}\right), \\ \Lambda_4 &= 2c\frac{E_0}{V^*(V^* - r)}, \\ \Lambda_5 &= -2c\frac{\varphi}{\eta}\left(F_0 - \frac{D_{\max}}{\eta}\right)\frac{1}{\eta - r}, \\ S(T) &= S_0 - WT - \frac{\varphi}{\eta}\left(D_{\max}T - \left(F_0 - \frac{D_{\max}}{\eta}\right)(1 - e^{\eta T})\right) - \frac{E_0}{V^*}(1 - e^{V^* T}).\end{aligned}$$

Once we have λ_S, we can plug it in expression (39) to obtain λ_E. Integrating the resulting expression gives

$$\lambda_E(t) = \frac{a}{r-V^*} - \frac{bE_0}{r-2V^*}e^{V^*t} + \frac{\Lambda_1}{r-V^*} + \frac{\Lambda_2}{V^*}e^{-r(T-t)}e^{-2V^*t}$$
$$+ \frac{\Lambda_3}{r-V^*}\left(t+\frac{1}{r-V^*}\right)+\frac{\Lambda_4}{r-2V^*}e^{V^*t}+\frac{\Lambda_5}{r-V^*-\eta}e^{\eta t}+K_E e^{(r-V^*)t},$$

where K_E denotes the integration constant. To determine K_E we use the transversality condition $\lambda_E(T) = 0$, and substitute its value in the above expression. The co-state variable $\lambda_E(t)$ then reads as in expression (40).

$$\begin{aligned}\lambda_E(t) &= (\Lambda_1+a)\frac{1}{r-V^*}(1-\Gamma(t)) - \frac{bE_0}{r-2V^*}\left(e^{V^*t}-e^{V^*T}\Gamma(t)\right)\\ &+ \frac{\Lambda_2}{V^*}[e^{-2V^*t}e^{-r(T-t)} - e^{-2V^*T}\Gamma(t)]\\ &+ \frac{\Lambda_3}{r-V^*}\left(t+\frac{1}{r-V^*}-\left(T+\frac{1}{r-V^*}\right)\Gamma(t)\right)\\ &+ \frac{\Lambda_4}{r-2V^*}(e^{V^*t}-e^{V^*T}\Gamma(t))+\frac{\Lambda_5}{r-V^*-\eta}(e^{\eta t}-e^{\eta T}\Gamma(t)),\end{aligned} \tag{40}$$

where $\Gamma(t) = e^{-(r-V^*)(T-t)}$.

The expression for (41) is valid for as long as $\lambda_E(t)E(t) > 0$ at all times. This is the case for $T \lesssim 40$. If the planning horizon is longer than that, then (41) does not hold at all times and there will be a switch in the optimal emissions policy.

Provided that $T \lesssim 40$, the optimal path for the shadow price of the forest stock $\lambda_F(t)$ can be obtained analogously and is given by expression (41).

$$\begin{aligned}\lambda_F(t) = \varphi\Bigg[&\left(-\frac{\Lambda_1}{r-\eta}+\frac{\Lambda_2}{\eta}\Psi(t)\right)(1-\Psi(t))-\frac{\Lambda_4}{r-\eta-V^*}(e^{V^*t}-\Psi(t)) \\ &- \frac{\Lambda_3}{r-\eta}\left(t+\frac{1}{r-\eta}-\left(T+\frac{1}{r-\eta}\right)\Psi(t)\right)-\frac{\Lambda_5}{r-2\eta}(e^{\eta t}-\Psi(t)e^{-\eta T})\Bigg],\end{aligned} \tag{41}$$

where $\Psi(t) = e^{-(r-\eta)(T-t)}$.

Appendix D: Switching time

If there is only one switch in the optimal policy (switch at time $\tilde{t}_V$) then the jump should be of the following type: First apply $V_{\max}$ $\forall t \in [0, \tilde{t}_V]$ and then apply $V_{\min}$ $\forall t \in [\tilde{t}_V, T]$. Applying $V_{\max}$ always brings greater yields in the short run than does $V_{\min}$ and if one optimizes using a positive discount rate, it is better to allocate emissions at the beginning of the planning horizon.

Recall that, in absence of switch, we have that $S(t)$ is given by (17). Whereas, when there is a switch, the optimal expression for $S(t)$ changes. We now have a two-part expression, one before the switch and another afterwards.

$$S(t)=S_0-tW-\frac{\varphi}{\eta}D_{\max}t-\frac{E_0}{V_{\max}}(1-e^{V_{\max}t})+\frac{\varphi}{\eta}\left(F_0-\frac{D_{\max}}{\eta}\right)(1-e^{\eta t}), \forall t\in[0,\tilde{t}_V),$$

$$S(t)=S(\tilde{t}_V)-\left(t-\tilde{t}_V\right)\left(W+\frac{\varphi}{\eta}D_{\max}\right)-\frac{E_0}{V_{\min}}e^{V_{\max}\tilde{t}_V}(1-e^{V_{\min}\left(t-\tilde{t}_V\right)})$$
$$+\frac{\varphi}{\eta}\left(F(\tilde{t}_V)-\frac{D_{\max}}{\eta}\right)e^{\eta\tilde{t}_V}(1-e^{\eta\left(t-\tilde{t}_V\right)}), \quad \forall t\in[\tilde{t}_V,T],$$

where

$$S(\tilde{t}_V)=S_0-\tilde{t}_VW-\frac{\varphi}{\eta}D_{\max}\tilde{t}_V-\frac{E_0}{V_{\max}}(1-e^{V_{\max}\tilde{t}_V})+\frac{\varphi}{\eta}\left(F_0-\frac{D_{\max}}{\eta}\right)(1-e^{\eta\tilde{t}_V}).$$

These expressions are straightforward to obtain considering that

$$E(t)=\begin{cases} E_0e^{V_{\max}t}, & \forall t\in[0,\tilde{t}_V] \\ E_0e^{V_{\max}\tilde{t}_V}e^{V_{\min}(t-\tilde{t}_V)}, & \forall t\in[\tilde{t}_V,T]. \end{cases}$$

Once we have $S(t)$, $\lambda_S(t)$ can be computed using the transversality condition from the salvage value function. Proceeding similarly as we did to obtain $\lambda_S(t)$ in the case without a switch, the following expression for the interval $[\tilde{t}_V,T]$ is obtained:

$$\lambda_S(t)=\Upsilon_1+\Upsilon_2e^{-r(T-t)}+\Upsilon_3t+\Upsilon_4e^{V_{\min}\left(t-\tilde{t}_V\right)}+\Upsilon_5e^{\eta t}, \tag{42}$$

where:

$$\Upsilon_1=-\frac{2c}{r}\left[S(\tilde{t}_V)-\underline{S}-\frac{E_0e^{V_{\max}\tilde{t}_V}}{V_{\min}}+\frac{\varphi}{\eta}\left(F(\tilde{t}_V)-\frac{D_{\max}}{\eta}\right)\right]$$
$$-\frac{2c}{r}\left(W+\frac{\varphi D_{\max}}{\eta}\right)\left(\tilde{t}_V-\frac{1}{r}\right),$$

$$\Upsilon_2=-\Upsilon_1-2c\left[\frac{W}{r}T+\frac{\varphi}{\eta}\left(\frac{D_{\max}}{r}T-\left(F(\tilde{t}_V)-\frac{D_{\max}}{\eta}\right)\frac{1}{\eta-r}e^{\eta(T-\tilde{t}_V)}\right)\right.$$
$$\left.+\frac{E_0}{V_{\min}}e^{V_{\max}\tilde{t}_V}\frac{e^{V_{\min}(T-\tilde{t}_V)}}{V_{\min}-r}\right]+2c\phi[\underline{S}-S(T)],$$

$$\Upsilon_3=\frac{2c}{r}\left(W+\frac{\varphi}{\eta}D_{\max}\right),$$

$$\Upsilon_4=2c\frac{E_0e^{V_{\max}\tilde{t}_V}}{V_{\min}\left(V_{\min}-r\right)},$$

$$\Upsilon_5=-2c\frac{\varphi}{\eta}\left(F(\tilde{t}_V)-\frac{D_{\max}}{\eta}\right)\frac{1}{\eta-r}.$$

Once $\lambda_S(t)$ $\forall t\in[\tilde{t}_V,T]$ is known, $\lambda_S(t)$ $\forall t\in[0,\tilde{t}_V]$ can be computed analogously and can be written in a compact manner as follows:

$$\lambda_S(t)=\Sigma_1+\Sigma_2e^{r\left(t-\tilde{t}_V\right)}+\Sigma_3t+\Sigma_4e^{V_{\max}t}+\Sigma_5e^{\eta t},$$

where

$$\begin{aligned}
\Sigma_1 &= -\frac{2c}{r}\left[S_0 - \underline{S} - \frac{E_0}{V_{\max}} + \frac{\varphi}{\eta}\left(F_0 - \frac{D_{\max}}{\eta}\right) - \left(\frac{W}{r} + \frac{\varphi}{\eta}\frac{D_{\max}}{r}\right)\right],\\
\Sigma_2 &= -\Sigma_1 - 2c\left[\left(\frac{W}{r} + \frac{\varphi}{\eta}\frac{D_{\max}}{r}\right)\tilde{t}_V - \frac{\varphi}{\eta}\left(F_0 - \frac{D_{\max}}{\eta}\right)\frac{e^{\eta\tilde{t}_V}}{\eta - r} + \frac{E_0}{V_{\max}}\frac{e^{V_{\max}\tilde{t}_V}}{V_{\max} - r}\right]\\
&\quad + \lambda_S(\tilde{t}_V),\\
\Sigma_3 &= \frac{2c}{r}\left(W + \frac{\varphi}{\eta}D_{\max}\right),\\
\Sigma_4 &= 2c\frac{E_0}{V_{\max}\left(V_{\max} - r\right)},\\
\Sigma_5 &= -2c\frac{\varphi}{\eta}\left(F_0 - \frac{D_{\max}}{\eta}\right)\frac{1}{\eta - r},
\end{aligned}$$

and where $\lambda_S(\tilde{t}_V)$ is the boundary condition to this problem and can be obtained by substituting for time $t = \tilde{t}_V$ in equation (42).

Now $\lambda_S(t)\ \forall t \in [\tilde{t}_V, T]$ and $\lambda_S(t)\ \forall t \in [0, \tilde{t}_V]$ are known; $\lambda_E(t)\ \forall t \in [\tilde{t}_V, T]$ and $\lambda_E(t)\ \forall t \in [0, \tilde{t}_V]$ can be obtained analogously. In this case it is easier since the boundary condition for λ_E (i.e., $\lambda_E(T)$) is equal to zero. The expression of $\lambda_E(t)\ \forall t \in [\tilde{t}_V, T]$ reads:

$$\begin{aligned}
\lambda_E(t) = &\ \frac{a}{r - V_{\min}} - \frac{bE_0 e^{V_{\max}\tilde{t}_V}}{r - 2V_{\min}} e^{V_{\min}\left(t - \tilde{t}_V\right)} + \frac{\Upsilon_1}{r - V_{\min}} - \frac{\Upsilon_2}{V_{\min}} e^{-r(T-t)}\\
&+ \frac{\Upsilon_3}{r - V_{\min}}\left(t + \frac{1}{r - V_{\min}}\right) + \frac{\Upsilon_4}{r - 2V_{\min}} e^{V_{\min}\left(t - \tilde{t}_V\right)}\\
&+ \frac{\Upsilon_5}{r - V_{\min} - \eta} e^{\eta\left(t - \tilde{t}_V\right)} + K_E e^{(r - V_{\min})t}.
\end{aligned}$$

The constant of integration K_E can be obtained using the transversality condition $\lambda_E(T) = 0$. Denote $\Pi(t) = e^{(r - V_{\min})(t - T)}$; then the value of $\lambda_E(t)\ \forall t \in [\tilde{t}_V, T]$ can be written as follows:

$$\begin{aligned}
\lambda_E(t) = &\ (a + \Upsilon_1)\frac{1}{r - V_{\min}}\left(1 - \Pi(t)\right) - \frac{\Upsilon_2}{V_{\min}}\left(e^{-r(T-t)} - \Pi(t)\right)\\
&- \frac{bE_0 e^{V_{\max}\tilde{t}_V}}{r - 2V_{\min}}\left(e^{V_{\min}\left(t - \tilde{t}_V\right)} - e^{V_{\min}\left(T - \tilde{t}_V\right)}\Pi(t)\right)\\
&+ \frac{\Upsilon_3}{r - V_{\min}}\left[\left(t + \frac{1}{r - V_{\min}}\right) - \left(T + \frac{1}{r - V_{\min}}\right)\Pi(t)\right]\\
&+ \frac{\Upsilon_4}{r - 2V_{\min}}\left(e^{V_{\min}\left(t - \tilde{t}_V\right)} - e^{V_{\min}\left(T - \tilde{t}_V\right)}\Pi(t)\right)\\
&+ \frac{\Upsilon_5}{r - V_{\min} - \eta}\left(e^{\eta\left(t - \tilde{t}_V\right)} - e^{\eta\left(T\tilde{t}_V\right)}\Pi(t)\right).
\end{aligned} \tag{43}$$

Similarly, for $\lambda_E(t)\,\forall\, t \in [0, \tilde{t}_V]$ we obtain the following expression:

$$\begin{aligned}
\lambda_E(t) = &\, (a+\Sigma_1)\,\frac{1}{r-V_{\max}}\,(1-\Delta(t)) - \frac{bE_0}{r-2V_{\max}}\left(e^{V_{\max}t} - e^{V_{\max}\tilde{t}_V}\,\Delta(t)\right) \\
&- \frac{\Sigma_2}{V_{\max}}\left(e^{-r(\tilde{t}_V-t)} - \Delta(t)\right) + \frac{\Sigma_4}{r-2V_{\max}}\left(e^{V_{\max}t} - e^{V_{\max}\tilde{t}_V}\Delta(t)\right) \\
&- \frac{\Sigma_3}{r-V_{\max}}\left[\left(t + \frac{1}{r-V_{\max}}\right) - \left(\tilde{t}_V + \frac{1}{r-V_{\max}}\right)\Delta(t)\right] \\
&+ \frac{\Sigma_5}{r-V_{\max}-\eta}\left(e^{\eta t} - e^{\eta\tilde{t}_V}\,\Delta(t)\right) + \lambda_E(\tilde{t}_V)\Delta(t), \qquad (44)
\end{aligned}$$

where $\lambda_E(\tilde{t}_V)$ in (44) can be obtained from (43) and $\Delta(t) = e^{(r-V_{\max})\left(t-\tilde{t}_V\right)}$.

With the two equations for $\lambda_E(t)$ (before and after the switch) the switching time can be obtained. The switching time (provided it is unique) has to satisfy the following first-order condition:

$$\begin{aligned}
\lambda_E(t)E(t) > 0 \quad \forall t \in [0, \tilde{t}_V), \\
\lambda_E(t)E(t) < 0 \quad \forall t \in (\tilde{t}_V, T].
\end{aligned}$$

References

[1] Andrés-Domenech, P., Saint-Pierre, P., & Zaccour, G. (2011). Forest conservation and CO_2 emissions: A viable approach. *Environmental Modeling & Assessment, 16*(6), 519-539.

[2] Bahn, O., Haurie, A., & Malhamé, R. (2008). A stochastic control model for optimal timing of climate policies. *Automatica, 44*, 1545-1558.

[3] Barbier, E. B., & Burgess, J. C. (2001). The economics of tropical deforestation. *Journal of Economic Surveys, 15*(3), 413-433.

[4] Barbier, E. B., & Rauscher, M. (1994). Trade, tropical deforestation and policy intervention. *Environmental and Resource Economics, 4*(1), 75-90.

[5] Benchekroun, H., & Long, N. V. (2002). On the multiplicity of efficiency-inducing tax rules. *Economics Letters, 76*, 331-336.

[6] Bernstein, P. M., Montgomery, W. D., & Tuladhar, S. D. (2006). Potential for reducing carbon emissions from non-Annex B countries through changes in technology, *Energy Economics, 28*(5-6), 742-762.

[7] Breton, M., Martín-Herrán, G., & Zaccour, G. (2006). Equilibrium investment strategies in foreign environmental projects. *Journal of Optimization Theory and Applications, 130*(1), 23-40.

[8] Breton, M., Zaccour, G. & Zahaf. M. (2005). A differential game of joint implementation of environmental projects. *Automatica, 41*, 1737-1749.

[9] Dockner, E. J., & Long, N. V. (1993). International pollution control: Cooperative versus noncooperative strategies. *Journal of Environmental Economics and Management, 24*, 13-29.

[10] Energy Information Administration (EIA) (2008). *International Energy Outlook 2008.*

[11] Eswaran, H., Lal, R., & Reich, P. F. (2001). Land degradation: an overview. In E. M. Bridges, I. D. Hannam, L.R. Oldeman, F. W. T. Penning de Vries, S. J. Scherr & S. Sombatpanit (Eds.), *Response to Land Degradation* (pp. 20-35). Enfield, NH, USA: Science Publishers Inc.

[12] Food and Agriculture Organization of the United Nations (FAO) (1997). *Asia-Pacific Forestry Sector Outlook Study: Forest Industry Structure and the Evolution of Trade Flows in the Asia-Pacific Region–Scenarios to 2010.*

[13] Food and Agriculture Organization of the United Nations (FAO) (2006). *Global Forest Resources Assessment 2005.*

[14] Food and Agriculture Organization of the United Nations (FAO) (2010). *Global Forest Resources Assessment 2010.*

[15] Fredj, K., Martín-Herrán, G., & Zaccour, G. (2004). Slowing deforestation Rate through subsidies: A differential game, *Automatica, 40*(2), 301-309.

[16] Fredj, K., Martín-Herrán, G., & Zaccour, G. (2006). Incentive mechanisms to enforce sustainable forest exploitation. *Environmental Modeling & Assessment, 11*(2),145-156.

[17] Intergovernmental Panel on Climate Change (IPCC) (2000). Land Use, Land-Use Change and Forestry. *Special report.* Cambridge: Cambridge University Press.

[18] Intergovernmental Panel on Climate Change (IPCC) (2007). *Climate Change 2007: Impacts, Adaptation and Vulnerability, Fourth Assessment Report.* Cambridge: Cambridge University Press.

[19] Jørgensen, S., Martín-Herrán, G., & Zaccour, G. (2010). Dynamic games in the economics and management of pollution. *Environmental Modeling & Assessment, 15*(6), 433-467.

[20] Le Quéré, C., Raupach, M. R., Canadell, J. G., Marland, G., et al. (2009). Trends in the sources and sinks of carbon dioxide. *Nature Geoscience, 2*, 831-836.

[21] Long, N. V. (1992). Pollution control: A differential game approach. *Annals of Operations Research, 37*, 283-296.

[22] Martín-Herrán, G., Cartigny, P., Motte, E., & Tidball, M. (2006). Deforestation and foreign transfers: A Stackelberg differential game approach. *Computers & Operations Research, 33*(2), 386-400.

[23] Martín-Herrán, G., Tidball, M. (2005). Transfer mechanisms inducing a sustainable forest exploitation. In C. Deissenberg & R. Hartl (Eds.) *Optimal Control and Dynamic Games: Applications in Finance* (pp. 85-103). *Management Science and Economics.* The Netherlands: Springer.

[24] National Oceanic & Atmospheric Administration (NOAA) (2007). *Trends in Atmospheric Carbon Dioxide.*

[25] Van Soest, D., & Lensink, R. (2000). Foreign transfers and tropical deforestation: What terms of conditionality? *American Journal of Agricultural Economics, 82*(2), 389-399.

[26] Van der Ploeg, F., & De Zeeuw, A. (1992). International aspects of pollution control. *Environmental & Resource Economics, 2*(2),117-139.

[27] Yeung, D. W. K., & Petrosjan, L. A. (2005). *Cooperative Stochastic Differential Games.* New York: Springer.

[28] Zaccour, G. (2008). Time consistency in cooperative differential games: A tutorial, *INFOR 46*(1), 81-92.