A Great Fish War Model with Asymmetric Players

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Abstract

This paper analyzes the coalitional Great Fish War model by assuming that players differ in their time preferences and use different discount rates. We derive the equilibrium payoffs of this coalitional game and investigate the impact of the asymmetry assumption on the extreme schemes of cooperative and non-cooperative equilibria. We then proceed to the computation of stable coalitions using time-consistent harvest sharing policies for the partial coordination scheme, in the case where players are divided into two groups (high and low discount rates). We find that asymmetry has an important impact on the way the resource is shared and on the profitability of coalitions. We also find that asymmetry is not a sufficient feature to overcome the puzzle of small coalitions.

1 Introduction

The Great Fish War model of Mirman (1979) is a parsimonious framework that has been extensively used for analyzing open-access problems in fisheries. A well known result is that the non-cooperative setting yields a prisoner's dilemma type of result, and that coordination of international fisheries is Pareto-improving (Levhari and Mirman 1980, Okuguchi 1981), raising the question of the stability of international coalitions.

Coordination in international fisheries has been mainly analyzed using a cooperative approach, comparing the full-coordination equilibrium with the no-cooperation equilibrium. The usual coordination instruments are trigger strategies (Hämäläinen *et al.* 1985, Cave 1987, Benhabib and Radner 1992), incentive strategies (Ehtamo and Hämäläinen 1993) and transfers (Pintassilgo *et al.* 2010, Mazalov and Rettieva 2010).

Partial coordination in the management of fisheries, where a subset of countries agree to coordinate their use of the resource, occupies a middle ground between full cooperation and independent exploitation, and is more consistent with what is observed in existing fishery management organizations.[1] Partial coordination has been analyzed using a non-cooperative approach (see Becker and Easter 1999, Kwon 2006 and Breton and Keoula 2012), requiring the cooperation agreement to be *self-enforcing*. As it is the case in environmental games (Barrett 1994) or in the cartel theory literature (d'Aspremont *et al.* 1983), stable large membership in partial coalitions cannot be obtained in the Great Fish War setting without the help of additional mechanisms, such as first-mover advantage (Kwon 2006) or farsightedness (Breton and Keoula 2012). Most partial coordination models assume that all players are identical, raising the question of the impact of the symmetry assumption in the puzzle of small coalitions.

One common source of asymmetry in the fishery economics literature is the marginal cost of fishing: the individual cost per unit of effort is different from one player to another. This kind of asymmetry is pervasive in analyzes relying on textbook models such as the basic one presented in Clark (1990). In an M-player setting, a recent example is the coalitional fishery game of Pintassilgo *et al.* (2010) addressing the resilience of Regional Fishery Management Organizations.

Another important source of asymmetry in fisheries is players' discount rates, translating in differing ways to evaluate the trade-off between immediate consumption and investment in the fish stock. However, different discount rates raise the problem of how to aggregate time preferences in a coalition. In a finite horizon setting, Gollier and Zeckhauser (2005) show that when individuals have heterogeneous constant rates of impatience, the group time preference will not be constant in general. In particular, exponential discounting yields a collective discount rate that decreases with the time horizon. Finding the cooperative equilibrium is then related to hyperbolic discounting (Laibson 1997) and gives rise to a time-consistency problem (see Fujii and Karp 2008 for a solution approach). Indeed, the cooperative solutions proposed by Munro (1979) and Plourde and Yeung (1989) are time inconsistent. Such "commitment solutions" see the share of the impatient player vanish with time, requiring a binding commitment by the players over an infinite horizon, which is not credible or politically feasible. Cooperative

[1] See for instance the Northwest Atlantic Fisheries Organization (NAFO - 12 members), the Western and Central Pacific Fisheries Commission (WCPFC - 25 members), and the International Commission for the Conservation of Atlantic Tunas (ICCAT - 47 members).

solutions of the Great Fish War with asymmetric players are derived in Houba *et al.* (2000) and Denisova and Garnaev (2008). Houba *et al.* (2000) solve a negotiation game between two players to find an acceptable sharing rule, and Denisova and Garnaev (2008) analyze the cooperative solution for a coalition of M players with equal sharing rule, however without adressing the question of coalition profitability.

This paper considers heterogeneous time preferences in a coalitional Great Fish War model involving M players. A first contribution is the characterization of profitable and Pareto-efficient time-consistent sharing rules among the members of a coalition. A second contribution is the derivation of the coalition versus fringe equilibrium strategies in a M-player Great Fish War coordination game, under both the simultaneous and sequential moves assumptions. Our results allow the comparison of the sizes of stable coalitions, steady-state stocks, catches and welfare under various scenarios.

The rest of the paper is organized as follows. Section 2 is a brief discussion of the assumption of heterogeneity of time preferences in fisheries. The Great Fish War model and assumptions are recalled in Section 3. In Section 4, we derive non-cooperative, cooperative and partial coordination solutions with both simultaneous and sequential moves, for the general case of M asymmetric players. Coalitional stability is studied in Sections 5 and 6: Section 5 examines the profitability of the grand coalition and characterizes efficient and acceptable sharing rules. Section 6 studies the stability of partial coalitions for a special case when the fishery involves two types of players characterized by contrasting discount rates, under both the simultaneous and sequential moves assumptions. Section 7 is a conclusion.

2 Time preference and discounting

This paper is based on a time-additive discounted utility model. The reference framework in the modeling of intertemporal decisions using discounted utilities has been popularised by Samuelson's (1937) work. It assumes the "same discount rate for all types of goods and all categories of intertemporal decisions."[2]

2.1 Heterogeneous discount rates

Because of the renewed interest over discounting issues, especially in relation with the climate change mitigation debate, there is a need to elaborate on the assumption of heterogeneous players in their discount rates. We start by the ubiquitous Ramsey rule for discounting consump-

[2] Frederick *et al.* (2002)

tion, expressing the discount rate r as

$$r = \rho - gc\frac{u^{''}(c)}{u^{'}(c)}$$

where c is the per capita consumption, u is the utility function, g is the growth rate of the economy or of consumption, and ρ is the pure time preference of agents. Accordingly, discount rates depend on pure time preference (explained by impatience and uncertainty about the future) and by expected wealth growth and decreasing marginal utility (later consumption will have less utility).

Many studies and academic papers have acknowledged a wide variation in the rates deemed appropriate to discount future utilities. Frederick *et al.* (2002) document the extent of disagreements found in studies aiming at measuring individuals' time discounting rates, which can span 0% through 70%. Many other authors, among whom Gollier and Zeckhauser (2005) and Jouini *et al.* (2010), have refined those arguments to pure time preference rates: in a deterministic setting and in the absence of any anticipation of future growth, rational individuals may disagree in their judgements about how to value today their own future utility as well as the welfare of future generations.

In the fishery economics literature, differing discount rates has been interpreted as divergences in the objectives of the management of the fishery. The higher the discount rate, the lower the proportion of biomass saved up for the following periods, and the higher the immediate consumption.

A first motivation for heterogeneous discount rates in a fishery is provided by Munro (1979): since there is a good deal of controversy over what constitutes the appropriate rate of discount for the management of fisheries, there is no reason why managers would agree on this rate of discount. Asymmetric access has also been proposed by Munro (1990) in a two-player setting referring to the Pacific Islands Tuna Fishery: due to a change in the fish runs, some of the islands found themselves in a much more favorable position for the access to the resource, and thus placed greater emphasis on future returns than the others, translating into lower rates of discount. A third motivating example is provided in Vallée and Guillotreau (2010) in the context of the negotiation of access rights for European Union fleets to fish in the waters of the ACP (African, Caribbean, Pacific) countries. The authors consider both the possibility of higher preferences for the future of the ACP countries, because of long-term ownership considerations, and the possibility of poor countries using high discount rates and showing short-term interest for their resources. Economic and size inequality is also used in Lohoues

(2006), who argues that competing fleets of different capacity may have different returns on investment, translating into different discount rates.

2.2 Social planning and altruism

Summarizing the position of most economists and philosophers on social rates of pure time preferences, Azar and Sterner (1996) assert that "there is no good ethical justification for using a pure rate of time preference larger than zero," which translates into giving less weight to the utility of future generations. Accordingly, in the literature of public policies to mitigate climate change, the level of ρ considered in the Ramsey rule is rather low: 1.5% in Nordhaus (2007) and 2% in Weitzman (2007). The Stern report (2007) picked $\rho = 0.1\%$ and the Green Book for appraising U.K. government projects proposes $\rho = 1\%$.[3] In that context, it is worthwile mentioning that Nowak (2006a, 2008) showed that the non-cooperative equilibrium solution in the Great Fish War model converges to an overtaking equilibrium when players' discount rate vanishes, with higher steady-state stock and consumption. This means that if agents would agree not to discount future returns, then they would all be better-off in the long term – but in the context of high seas fisheries, it seems that many social planners are not willing to put equal weights on immediate and future consumption, and are effectively using different discount rates.

Competition between successive generation for the exploitation of a resource can also be modeled using multigeneration models. In that case, altruism for future generations (possibly decreasing with time) translates into hyperbolic discounting and heterogeneous discount rates (see Nowak 2006b for an application to fisheries).

3 Model and assumptions

We recall the bioeconomic model underlying the fishery coalition game consisting in a welfare function for the fishery owner, and a growth function for the fish stock. The one-period utility function of the fishery owner is assumed logarithmic:

$$u(x) = \log x,$$

where x it the catch, or harvest. It is well known that the logarithmic function implies the absence in the model of intertemporal utility substitution effects.

[3] As reported by Jouini *et al.* (2010). One can also notice in these two studies the assumption of a logarithmic utility function, as in the fish war game.

The one-period growth function of the non-harvested fish stock is given by the relation

$$s_{t+1} = s_t^{\alpha},\ 0 < \alpha < 1,$$

where s is the level of the fish stock and where the saturation level of the biomass is normalized to one. Constant α is interpreted as the (inverse) growth potential of the fish stock:[4] the smaller is α (closer to 0), the higher is the regeneration capacity of the fish stock, and for α close to 1, the resource is almost non-renewable. Harvesting reduces the fish stock; we call *residual stock* the amount of fish available for growth into the next period.

Denote $V_{[xy]} : \mathbb{R}^{+} \rightarrow \mathbb{R}$ the total discounted utility of the fishery owner over an infinite horizon, as a function of the fish stock, if the amount of her harvest is given by the function x and the residual stock is given by the function y.

The value function $V_{[xy]}$ satisfies the recursion

$$V_{[xy]}(s) = \log x + \delta V_{xy}\left(y^{\alpha}\right) \tag{1}$$

where δ is the one-period discount factor, $0 \leq \delta < 1$.

A useful result from the symmetric Great Fish War model is recalled as follows (see Breton and Keoula 2012):

Proposition 1 *If the catch and residual stock are both stationary linear functions of the fish stock,*

$$\begin{aligned} x(s) &= hs \\ y(s) &= qs, \end{aligned}$$

then the value function takes the log-linear form $V_{[xy]}(s) = A + \frac{1}{1-\alpha\delta} \log s$, *where*

$$A = (1-\delta)^{-1} \left(\log h + \frac{\alpha\delta}{1-\alpha\delta} \log q \right).$$

Our coalition game is based on four assumptions:

A1 Players have different time preferences, δ_i, $i = 1, ..., M$,

A2 only one coalition forms,

A3 there is no transfer of harvest between coalition members, and

[4] See Lewis and Cowens (1982).

A4 players in a coalition are characterized by their strategic importance, γ_i, $i = 1, ..., M$.

The second assumption is in line with the 1995 revised United Nations Law of the Sea, which admonishes countries to cooperate in the management of the high seas fisheries, without precluding any interested country to have access to them (see Bjorndal *et al.* 2000). This assumption is also consistent with existing fishery management organizations.

The third assumption rules out a transfer mechanism as a way to sustain cooperation. This reinforces the pure utility form of the individual payoffs, casting any cooperative scheme as a cooperative game of asymmetric players with non-transferable payoffs. Players in a coalition only negotiate a set of cooperative fishing strategies, and each player then consumes her own catch over time.

The fourth assumption is used to recognize that a coalition may give different weights to players in its global objective and its implication will be discussed further in Section 5. We now recall the cooperative and non-cooperative solutions of the Great Fish War model with M asymmetric players.

4 Equilibrium solutions with asymmetric players

The fishery is populated by M competing countries (players), where Player i is characterized by her discount factor δ_i and strategic importance γ_i. Define the parameters

$$\beta_i \equiv \frac{\alpha\delta_i}{1-\alpha\delta_i}, i = 1, ..., M.$$

Notice that each $\beta_i : (0, 1) \to \mathbb{R}^+$ is an increasing function of the parameter $\alpha\delta_i$. We derive in the following sections the non-cooperative, cooperative and partial coordination equilibrium solutions, where h_i^E, H^E and q^E characterize respectively the individual harvest, total harvest and residual stock corresponding to equilibrium solution E. Detailed developments are provided in Appendix 8.1.

4.1 The non-cooperative case

Assuming that a dynamic non-cooperative equilibrium exists, and that each player's value function takes the log-linear form

$$V_i^N(s) = A_i^N + (1 + \beta_i)\log s, \ \ i = 1, ..., M,$$

one can show (see Appendix 8.1.1) that the corresponding equilibrium fishing rule of each player and the residual stock are both stationary linear functions of the current stock. Using Proposition 1, this verifies the assumption about the form of the value functions. Denoting

$$b \equiv \sum_{i=1}^{M} \beta_i^{-1},$$

we obtain

$$A_i^N = (1-\delta_i)^{-1} \left(\log h_i^N + \beta_i \log q^N\right), \; i = 1, ..., M$$

where

$$\begin{aligned} h_i^N &= \frac{1}{\beta_i (b+1)}, \; i = 1, ..., M \\ H^N &= \frac{b}{b+1} \\ q^N &= \frac{1}{b+1}. \end{aligned}$$

One can get some interpretation of the b and β_i's by drawing a parallel to the valuation of growing perpetuities. The fish stock is a rental asset with a constant inverse growth potential α. Notice that the inverse growth potential affects the current fish stock exponentially, unlike a growth rate which would have had a multiplicative effect. Hence, it appears that $\alpha\delta_i$ is a modified discount factor. The corresponding discount rate is thus $\frac{1-\alpha\delta_i}{\alpha\delta_i}$, equal to β_i^{-1}, which can be termed "bionomic[5] discount rate" while β_i is the "bionomic multiplier".

Notice that players with lower discount rates (more patient) always have a smaller non-cooperative payoff than players with a higher discount rate (see Appendix 8.1.1).

Claim 1 *In the fully non-cooperative solution, the total harvest is directly related to the average of the bionomic discount rates, while each player's share of the total harvest is proportional to her own bionomic discount rate; as a consequence, patient players (with lower discount rates) always harvest less than impatient players, and their long-term payoff is lower.*

4.2 The cooperative solution

The cooperative solution of the Great Fish War is obtained by first solving for a Pareto efficiency condition. Denoting V_i^C $(i = 1, ..., M)$

[5] The qualifier is used after Clark (1990, p. 28), for situations determined by both biological and economic parameters.

the individual payoff function of a cooperative player, the identification of Pareto efficient outcomes boils down to choosing weights $\gamma_i > 0$, $i = 1, ..., M$, and a corresponding joint harvesting strategy x that solve the maximization problem (see Leitmann 1974, Yeung and Petrosyan 2006):

$$\max_x \left\{ \sum_{i=1}^{M} \gamma_i V_i^C \right\}.$$

The γ_i's are strategic weights, in the sense that their choice reflects the relative importance given to the payoffs of the different players in the joint management of the resource. We assume that the γ_i's are constant over time.[6] These weights could be, for instance, the result of prior negotiations between the members of the coalition.

In deciding of a joint management strategy, the coalition chooses both the total catch and the way to divide it among members. Assume that the value function of each player when using the joint optimal strategy takes the log-linear form:

$$V_i^C(s) = A_i^C + (1 + \beta_i) \log s.$$

The total payoff of the coalition then satisfies:

$$\begin{aligned} &\sum_{i=1}^{M} \gamma_i \left(A_i^C + (1 + \beta_i) \log s \right) \\ &= \max_{X,\lambda} \left\{ \sum_{i=1}^{M} \gamma_i \left(\log \lambda_i X + \delta_i A_i^C + \beta_i \log(s - X) \right) \right\} \end{aligned} \tag{2}$$

where $X = \sum_{i=1}^{m} x_i$ is the total catch of the coalition and where the vector λ indicates the way this catch is shared among members, $x_i = \lambda_i X$. Again, one can show (see Appendix 8.1.2) that optimal fishing strategies and residual stock are linear in the stock, thus verifying the assumption on the form of the value function. Denoting

$$G \equiv \sum_{i=1}^{M} \gamma_i$$

$$B \equiv \frac{\sum_{i=1}^{M} \gamma_i \beta_i}{G},$$

[6] Since we assume that the coalition solves an infinite horizon problem, it makes sense to assume that weights are constant over time. We will show below that profitability for each member of the coalition can be sustained with constant weights over the infinite horizon of the game.

we obtain

$$A_i^C = (1-\delta_i)^{-1}\left(\log h_i^C + \beta_i \log q^C\right),\ i = 1, ..., M$$

where

$$\begin{aligned} h_i^C &= \frac{\gamma_i}{G}\frac{1}{B+1}, \quad i = 1, ..., M \\ H^C &= \frac{1}{B+1} \\ q^C &= \frac{B}{B+1}. \end{aligned}$$

Under cooperation, the total harvest is inversely related to the parameter B, which is a weighted average of the players' bionomic multipliers. This result holds for any sharing vector λ: whatever the way a coalition divides the quantity harvested, a coalition giving higher weights to impatient players will harvest larger amounts for immediate consumption. However, it is optimal for the coalition to set the sharing vector so that λ_i is proportional to γ_i, $i = 1, ..., M$. It is interesting to note that the individual catch of players is then not necessarily related to their time preference, and that it is no longer the case that more patient players harvest less than impatient ones. For instance, if players in a coalition have equal strategic weights, then they all harvest the same amount, each player having to bear with the others that may be more or less patient than her. However, if two players have the same strategic weight, then it is still the case that the payoff of the more patient player is lower (see Appendix 8.1.2).

Notice that parameters b and B satisfy $bB > 1$:

$$b = \sum_{i=1}^{M} \frac{1}{\beta_i} > \frac{1}{\beta_{\min}} > \frac{1}{B}, \tag{3}$$

where $\beta_{\min}$ is the multiplier of the less patient player. As a consequence, irrespective of the weight vector, and even if all the strategic weight is given to the less patient player, the total harvest is smaller under cooperation than under competition:

$$\begin{aligned} H^C - H^N &= \frac{1}{B+1} - \frac{b}{b+1} \\ &= -\frac{bB-1}{(B+1)(b+1)}. \end{aligned}$$

Since the steady-state of the fish stock is given by $\bar{s} = q^{\frac{\alpha}{1-\alpha}}$, we then recover the classical result that the steady state of the fish stock under cooperation is larger than the steady state in the pure Nash game (Levhari and Mirman, 1980).

Claim 2 *In the fully cooperative solution, the total harvest is inversely related to the average of bionomic multipliers, weighted by strategic weight, while each player's share of the total harvest is proportional to her strategic weight in the coalition. For any strategic weight distribution among cooperating players, the quantity harvested is smaller under cooperation than under competition. For equal strategic weigths, patient players (with lower discount rates) have lower long-term payoffs.*

At this point, it is interesting to discuss the time consistency issue. Notice that in defining the optimization problem of the coalition in (2), we are implicitly seeking (feedback) Markov-perfect strategies. Maximizing the total payoff of the coalition over an infinite horizon in the space of (open-loop) time-dependent strategies will yield a higher payoff – but a time inconsistent solution. This is illustrated in Figure 1 below where the optimal time-dependent harvesting strategy of a coalition of two players with equal strategic weight is represented over time. We see that initially, both players harvest the same amount, but the quantity harvested by the less patient player (Player 1) is vanishing over time. This solution is only implementable if the coalition makes a binding commitment not to re-optimize the total welfare at any time over the infinite horizon; indeed, if the coalition reconsiders its total welfare at the end of the first year of operation, the optimal solution at that time will again prescribe that both players harvest the same amount, violating the initial commitment.

This time inconsistency in a deterministic setting stems from the fact that the discount factor of a "representative agent" having the same utility function as the coalition is not a constant. Indeed, the discount factor δ representing the coalition's aggregate time-preference satisfies

$$\begin{aligned} &\sum_{i=1}^{M} \gamma_i \left(\log\left(h_i^C s\right) + \delta\left(A_i^C + (1+\beta_i)\log\left(\left(q^C s\right)^{\alpha}\right)\right)\right) \\ =&\sum_{i=1}^{M} \gamma_i \left(A_i^C + (1+\beta_i)\log s\right), \end{aligned}$$

or equivalently

$$\delta(s) = \frac{\sum_{i=1}^{M} \gamma_i \left(A_i^C + (1+\beta_i)\log s - \log\left(h_i^C s\right)\right)}{\sum_{i=1}^{M} \gamma_i \left(A_i^C + (1+\beta_i)\log\left(q^C s\right)^{\alpha}\right)},$$

where it is apparent that the aggregate discount factor is a function of the fish stock. Figure 2 illustrates this relation for the example illustrated in Figure 1.

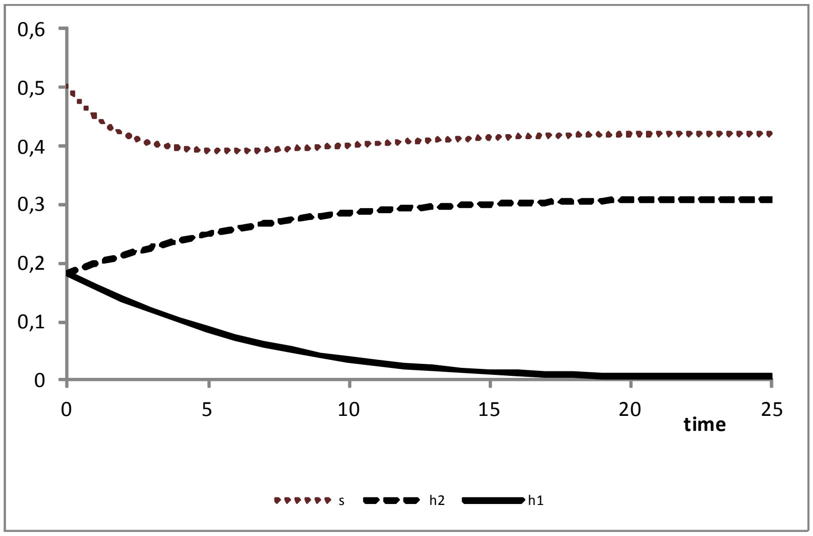

Figure 1: Time-inconsistent harvesting strategy of a coalition of two players. Parameter values are $\alpha = 0.7$, $\delta_1 = 0.8$, $\delta_2 = 0.99$, $\gamma_1 = \gamma_2 = 1$.

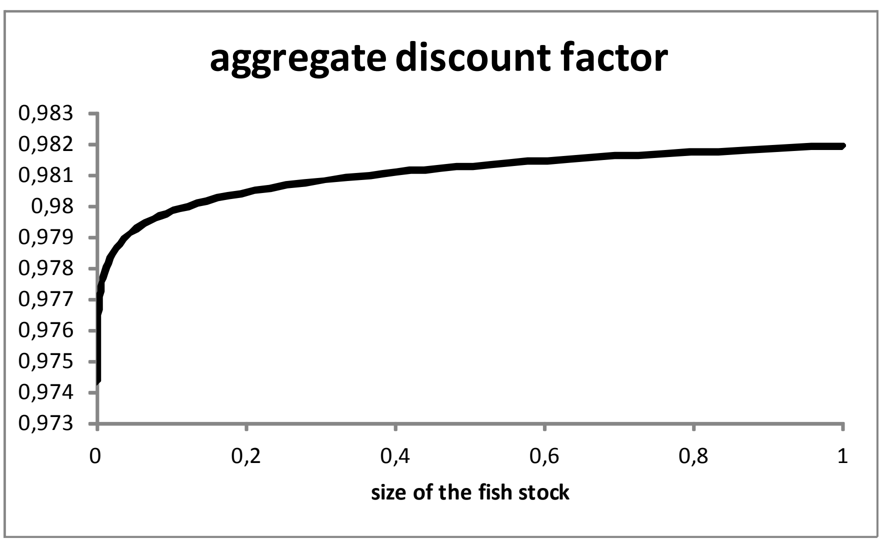

Figure 2: Aggregate discount factor as a function of the fish stock for a coalition of $n = 2$ heterogeneous players. Parameter values are $\alpha = 0.7$, $\delta_1 = 0.8$, $\delta_2 = 0.99$.

4.3 Partial coordination: a coalition versus a fringe

Now assume that the fishery is composed of a coalition of m players and of n outsiders, where $m+n = M$. Players of types $\delta_1, \delta_2, ..., \delta_m$ cooperate and maximize a weighted average of their discounted payoff. Players of types $\delta_{m+1}, \delta_{m+2}, ..., \delta_M$ do not cooperate and maximize their individual payoffs, given the strategy of the other players. Denote

$$G_m = \sum_{i=1}^{m} \gamma_i, \quad G_n = \sum_{i=m+1}^{m+n} \gamma_i,$$
$$B_m = \frac{\sum_{i=1}^{m} \gamma_i \beta_i}{G_m}, \quad B_n = \frac{\sum_{i=m+1}^{m+n} \gamma_i \beta_i}{G_n},$$
$$b_m = \sum_{i=1}^{m} \frac{1}{\beta_i}, \quad b_n = \sum_{i=m+1}^{m+n} \frac{1}{\beta_i},$$

where the (litteral) subscripts m and n indicate that the summations correspond to the first m or to the last n players respectively. Notice that parameters corresponding to the same set of players satisfy (3), so that $b_m B_m > 1$ and $b_n B_n > 1$. Using the results of the two preceding sections, it is easy to show that the total catch of the coalition when the outsiders are harvesting a total of Y is given by

$$X = \frac{s - Y}{B_m + 1}. \tag{4}$$

In the same way, the individual catch of outsider i when the coalition is harvesting a total of X is given by

$$y_i = \frac{1}{\beta_i} \frac{s - X}{b_n + 1}, i = m + 1, ..., M$$
$$Y = \sum_{i=m+1}^{M} y_i = \frac{b_n}{b_n + 1} (s - X). \tag{5}$$

Accordingly, we distinguish two possibilities for the hierarchy of players' moves.

4.3.1 Partial coordination with simultaneous moves

In this case, the coalition and the outsiders announce their harvesting strategies simultaneously, and the equilibrium concept is based on a Nash game between the $n+1$ players. Simultaneous solution of (4)-(5) yields:

$$X = \frac{s}{B_m(b_n + 1) + 1}$$
$$Y = \frac{b_n B_m}{B_m(b_n + 1) + 1} s,$$

so that the equilibrium fishing strategies and the residual stock are linear in the stock, thus verifying that value functions are log-linear. We finally obtain

$$A_i^S = (1-\delta_i)^{-1} \left(\log h_i^S + \beta_i \log q^S\right),\ i = 1, ..., M,$$

where

$$\begin{aligned} h_i^S &= \frac{\gamma_i}{G_m} \frac{1}{B_m (b_n + 1) + 1}, i = 1, ..., m \\ h_i^S &= \frac{1}{\beta_i} \frac{B_m}{B_m (b_n + 1) + 1}, i = m+1, ..., M \\ H^S &= \frac{1 + B_m b_n}{B_m (b_n + 1) + 1} \\ q^S &= \frac{B_m}{B_m (b_n + 1) + 1}. \end{aligned}$$

Notice that we recover the fully competitive case when $m = 1$ and the fully cooperative case when $m = M$. Comparing the residual stock according to the coalition structure, we obtain (see Appendix 8.1.3) $q^N < q^S < q^C$.

Claim 3 *In the partial coordination with simultaneous moves solution, the total harvest is directly related to the total of the bionomic discount rates of the outsiders, and inversely related to the weighted average of the bionomic multipliers of cooperating players. Members of the coalition share their total catch in proportion of their strategic weight, while outsiders share their total catch in proportion of their bionomic discount rate. The total harvest of players is larger than under full cooperation, smaller than under full competition.*

It is interesting to note that, if they have identical discount factors, say δ_1, an outsider is always faring better than a coalition member, irrespective of her strategic weight in the coalition:

$$\frac{B_m}{\beta_1} = \frac{\gamma_1}{G_m} + \frac{\sum_{i=2}^m \gamma_i \beta_i}{G_m \beta_1} > \frac{\gamma_1}{G_m}.$$

Moreover, for any partial coalition of m players where player i is an outsider, the payoff of player i is larger than what she would get in the non-cooperative case, since both the catch and the residual stock are larger (see Appendix 8.1.3), so that

$$A_i^S > A_i^N. \tag{6}$$

4.3.2 Partial coordination with first mover advantage

In this case, we assume that the coalition announces the harvesting strategies of member countries (e.g. in the form of quotas), and the outsiders decide on their own harvesting levels considering the announcement of the coalition. The equibrium concept is then based on a Stackelberg game where the leader is the coalition, taking the reaction of the outsiders to its announcement into account when determining its harvesting strategy. In that case, the optimization problem of the coalition becomes

$$\max_{X,\lambda}\left\{\sum_{i=1}^{m}\gamma_i\left(\log\lambda_i X+\delta_i A_i^F+\beta_i\log(s-X-\frac{b_n}{b_n+1}(s-X))\right)\right\}.$$

Solving this optimization problem, we again find that the equilibrium fishing strategies and the residual stock are linear in the stock, and finally obtain (see 8.1.4):

$$A_i^F=(1-\delta_i)^{-1}\left(\log h_i^F+\beta_i\log q^F\right),\ i=1,...,M,$$

where

$$\begin{aligned}
h_i^F &= \frac{\gamma_i}{G_m}\frac{1}{B_m+1}, i=1,...m,\\
h_i^F &= \frac{1}{\beta_i}\frac{B_m}{(B_m+1)(b_n+1)},\ i=m+1,...,M\\
H^F &= \frac{b_n(B_m+1)+1}{(B_m+1)(b_n+1)}\\
q^F &= \frac{B_m}{(B_m+1)(b_n+1)}.
\end{aligned}$$

Notice that each coalition member harvests more and each outsider harvests less than they would have in the simultaneous moves case (see 8.1.4). Comparing the residual stock according to the coalition structure, we obtain

$$q^F<q^S<q^C,$$

while the residual stock q^F can be higher or lower than in the case of full competition:

$$q^N-q^F=\frac{b_n+1-B_m b_m}{(B_m+1)(b_n+1)(b_m+b_n+1)},$$

where $B_m b_m>1$. If the number of coalition members is low (b_m is small with respect to b_n), then it may happen that partial coordination results in a lower residual stock than full competition. On the other hand, it is no longer necessarily true that an outsider is better-off than a coalition member having the same characteristics.

Claim 4 *In the partial coordination with first mover advantage, total harvest is directly related to the total of the bionomic discount rates of the outsiders, and inversely related to the weighted average of the bionomic multipliers of cooperating players. Players of both groups share their total catch in the same proportion as in the simultaneous moves case. However, when a coalition has first mover advantage, coalition members harvest more, outsiders harvest less, and the total harvest is higher than in the case of simultaneous moves.*

5 Coalition profitability

We now investigate coalition profitability, as defined by Carraro and Marchiori (2003). A profitable coalition is Pareto-improving: each cooperating player gets a payoff at least as high as what she would get when no coalition is formed, and at least one cooperating player has a higher payoff than when no coalition is formed. Profitability of the grand coalition has already been established in the Great Fish War model with symmetic players. We now proceed to check if these results still hold with asymmetric players. This amounts to checking if the following relation holds:

$$A_i^C + (1 + \beta_i) \log s \geq A_i^N + (1 + \beta_i) \log s, i = 1, ..., n$$

or equivalently:

$$\log h_i^C + \beta_i \log q^C \geq \log h_i^N + \beta_i \log q^N, i = 1, ..., n, \tag{7}$$

which highlights the trade-off between immediate consumption and investment made by players according to their discount factor, where at least one of these inequalities is strict. Notice that the profitability condition is independent of the resource level. A first obvious remark is that profitability is not guaranteed against any weight vector γ: clearly, when the strategic weight of one of the players vanishes, condition (7) cannot be satisfied.

5.1 Existence of acceptable weight vectors

Up to now, we assumed that players were characterized by their strategic importance, and that this parameter was constant over time. We saw that the vector γ of strategic weights defines both the total catch of the coalition and the share of each player. Since profitability is a necessary condition for a coalition to form, it can be assumed that profitability should play a role in the determination of strategic weights, for instance if these weights are the outcome of a bargaining process prior to the formation of the coalition. Such a bargaining game is considered for instance in Houba *et al.* (2010), where two countries with different time preferences engage in an alternating offer bargaining game for the determination of each countrie's fishing quota, which is assumed binding and everlasting once agreed-upon. The following proposition shows that it is always possible for a coalition to achieve profitability by agreeing on a set of constant weights.

Proposition 2 *There exists a vector γ such that profitability of the grand coalition is satisfied for any value of the fish stock.*

Proof. Consider a coalition of two players and the set of normalized weight vectors such that $\gamma_1 + \gamma_2 = 1$. Observe that Player 1's payoff is increasing in her own weight, from $-\infty$ when $\gamma_1 \to 0^+$, and that setting $\gamma_1 = 1$ gives Player 1 a strictly higher payoff than what she would get in the competitive case (using $bB > 1$ and $B = \beta_1$) :

$$\begin{aligned} A_1^N &= \log \frac{1}{\beta_1} \frac{1}{b+1} + \beta_1 \log \left(\frac{1}{b+1} \right) \\ &< \log \frac{1}{\beta_1} \frac{1}{\frac{1}{\beta_1}+1} + \beta_1 \log \left(\frac{1}{\frac{1}{\beta_1}+1} \right) \\ &= \log \frac{1}{B+1} + \beta_1 \log \left(\frac{B}{B+1} \right) = A_1^C \end{aligned}$$

As a consequence, it is always possible to find a value $\gamma_1^* \in (0,1)$ that makes the payoff of Player 1 equal to what she would get in the competitive case, so that (7) is satisfied for Player 1. Set $\gamma_2^* = 1 - \gamma_1^*$. Now, optimizing the weighted payoff of the coalition using γ^* will necessarily result in a solution that realizes a collective weighted outcome that is strictly higher than the collective weighted outcome corresponding to the non-cooperative solution, and consequently (7) is also satisfied for Player 2 with a strict inequality, since the payoff of Player 1 is the same. Finally, notice that the profitability condition (7) is independent of the level of the resource. This implies that the cooperative solution using a weight vector satisfying (7) is profitable for any value of the fish stock, and consequently Pareto-improving for all players over the infinite horizon of the game. This argument is extended to the M-player setting by considering sucessively larger subset of players with a collective weight γ_1. ■

Proposition 2 is illustrated in Figure 3 representing a situation involving five players divided into two groups according to their discount factors, where two impatient players (Group 1) have a discount factor of 0.8 and three patient players (Group 2) have a discount factor of

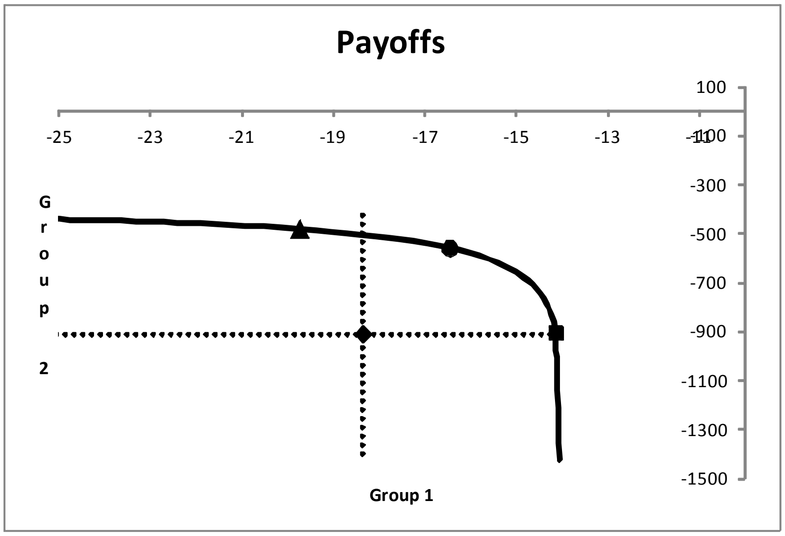

Figure 3: Efficient frontier and threat point for a coalition of 5 players, where $\alpha = 0.9$, $\delta_1 = \delta_2 = 0.8$ and $\delta_i = 0.99$, $i = 3, 4, 5$. The competitive solution is found where the two dotted lines cross.

0.99. Giving players with identical discount factors the same strategic weight, the solid line graphs the payoff of the players in Group 2 as a function of the payoff of the players in Group 1 as γ_1 varies from 0 to 1 (the Pareto-efficient frontier). The dotted lines represent each player's requirement to achieve the profitability condition (7), crossing at the competitive solution. The circle and square points on the efficient frontier represent weight vectors achieving profitability, while the triangle represents a weight vector that is unacceptable to players in Group 1.

5.2 Choice of a strategic weight vector

While Proposition 2 shows the existence of weight vectors such that the grand coalition is profitable, it does not point to an easy way to choose such a vector. If we assume that the choice of a weight vector is the result of a negotiation among the coalition members, then weight vectors can be characterized as the solution of some bargaining problem, where the threat point is the competitive solution. It is interesting to note that, since in the competitive case patient players have lower payoffs, patient players, attaching more importance to the preservation of the resource, have inferior bargaining positions, and consequently their strategic weight is expected to be lower in a profitable coalition.

For instance, the square point identified on Figure 3, with $\gamma_1 = 0.4914$ and $\gamma_2 = 0.0058$, corresponds to the Nash bargaining solution, and is found by requiring that the difference between the cooperative and non-cooperative outcomes for each player be equal. Other feasible points may be found by maximizing the product or the sum of these differences. The solution of the 2-player bargaining game in Houba *et al.* (2010) consists in giving all the benefits of cooperation to the first proposer, so that the second player gets exactly her non-cooperative payoff (this would correspond to either (0.2515, 0.1656) or (0.4917,0.0055) in the example illustrated in Figure 3). The drawback is that the determination of an acceptable weight vector in all these possibilities requires the numerical solution of systems of non-linear equations, involving the time preferences of all the players participating in the coalition.

In the context of partial coordination, which is more likely to be the norm in international fisheries, it is more realistic to assume that the share of a participating player in a coalition is fixed in some way that does not depend on the composition of the coalition. This allows to suppose that a coalition can enlarge (or reduce) without having each time to re-negotiate the strategic weights of all its members. In that direction, we will specifically consider two weighing rules that are easy to implement and to justify. In the first scenario, all players in the coalition have the same strategic weight, so that the coalition members agree to share the total catch equally among themselves. In the second scenario, strategic weights are inversely proportional to the players' bionomic multipliers, thus giving less strategic weight to the more patient players. Put differently, the coalition members agree to share the total catch in the same way as in the noncooperative solution, so that the impatient players harvest more of the ressource than the others in each time period. Both sharing rules are represented in Figure 3: the triangle corresponds to the equal weighting rule (0.2,0.2), and the circle corresponds to the proportional weighing rule (0.34,0.11). For this set of players, the grand coalition would not be profitable under the equal weighing rule, but would be under the proportional weighing rule.

When players have equal strategic weights, profitability may be difficult to attain if time preferences of the players are too far apart. For instance, specializing to a set of two groups of asymmetric players characterized by β_1 and β_2, of equal sizes $M_1 = M_2 = \frac{M}{2}$, condition (7) boils down to:

$$\log\left(\frac{2\beta_i}{M(\beta_1+\beta_2)}\right) \geq (1+\beta_i)\log\left(\frac{2\beta_1\beta_2(\beta_1+\beta_2+2)}{(\beta_1+\beta_2)(M\beta_1+M\beta_2+2\beta_1\beta_2)}\right), i = 1, 2.$$

Figure 4 illustrates the solution to this system of inequalities for $M = 10$.

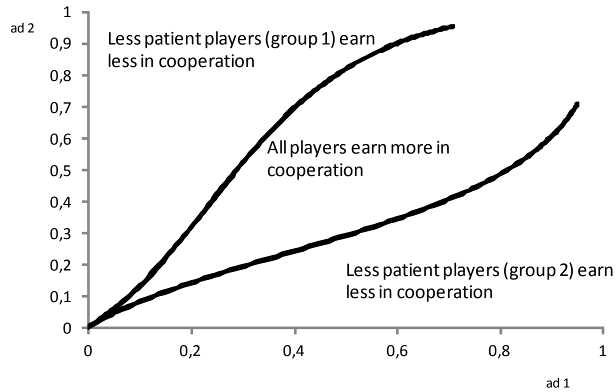

Figure 4: Comparison of payoffs according to parameter values for equal weights and $M = 10$.

We see that the parameter space is divided into three regions; cooperation is Pareto-improving for both groups of players when their discount factors are close enough. Otherwise, when the discrepancy between their discount factors is very high, only the players with the highest discount factor (the more patient ones) have a higher payoff under cooperation than under competition.

In the second scenario, where weights are given by $\gamma_i = \frac{1}{\beta_i}$, so that $Bb = M$, condition (7) translates to

$$(1 + \beta_i) \log \frac{B + M}{B + 1} \geq \log M.$$

Figure 5 illustrates this condition for $M = 5$: cooperation is not Pareto-improving for players characterized by β's that are sufficiently lower than the group's weighted average. This would happen when there is a significant discrepancy between time preferences, and when the group of impatient players is relatively smaller. For example, the point identified on Figure 5 corresponds to four players with $\beta_2 = 8.91$ in coalition with one player with $\beta_1 = 2.25$, so that $B = 5.6$; even if the impatient player receives 49.8% of the total catch while the others each receive 12.5% of it, her total discounted utility is less under cooperation than under competition. Notice however that rather extreme parameter values are needed for this to happen (with $\alpha = 0.9$, discount rates are respectively 30% and 0.1% in the above example).

In the second scenario, it is easy to show that all members of the coalition harvest less than what they would in the noncooperating case, but this is no longer true if we assume that coalition members have equal

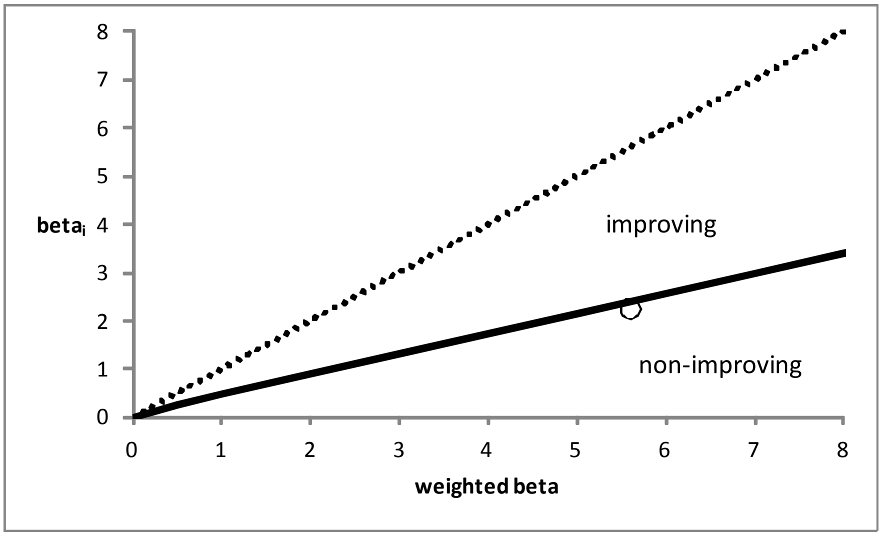

Figure 5: Region where cooperative solution is not Pareto-improving for $M = 5$. Player i's welfare is lower under cooperation when β_i is below the solid line. The dotted line is $\beta_i = B$.The circle corresponds to $\alpha = 0.9$, $m_1 = 1$, $m_2 = 4$, $\delta_1 = 0.769$, $\delta_2 = 0.999$.

strategic weights, even in profitable coalitions. In that case, when the discrepancy between discount factors is relatively high, the more patient players harvest more in cooperation, because they have to bring their capture up to the common level of all players (see Figure 6).

6 Coalition stability

We now examine the stability of partial coalitions. Following d'Aspremont et al. (1983), sufficient conditions for stability of a coalition consist in two conditions: *internal stability,* which requires no unilateral defection temptation from individual coalition members, and *external stability*, which requires that outsiders not be tempted to unilaterally join the coalition.

We study a special case where there are only two types of players, and where each player is characterized by its discount factor and strategic weight (we implicitly assume that strategic weights of players with the same time preference are the same). Thus, M_1 players use a discount factor δ_1 and have a strategic weight γ_1, and M_2 players use a discount factor δ_2 and have a strategic weight γ_2 where $M_1 + M_2 = M$. The coalition is of size m such that $m = m_1 + m_2$, with m_1 (respectively m_2) players of types δ_1 (respectively δ_2). Outsiders of each type are in numbers n_1 and n_2. Payoffs of players depend on the size and composition

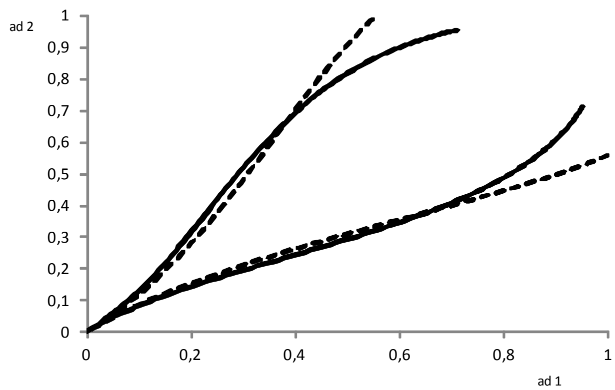

Figure 6: An illustration with two groups of 5 players and equal weighing rule. Profitable coalitions happen for parameter values inside the solid lines. Cooperating players harvest less than in the competitive case for parameter values inside the dotted lines.

of the coalition, and we now denote

$$V_{ki}(s; m_1, m_2) = A_{ki}(m_1, m_2) + (1 + \beta_i) \log s$$

the payoff at s of a player of type i who is an outsider ($k = o$) or an insider ($k = c$) when the coalition is composed of m_1 members of type 1 and m_2 members of type 2. Internal and external stability conditions can then be expressed as

$$\left.\begin{array}{l} A_{c1}(m_1, m_2) > A_{o1}(m_1 - 1, m_2),\ m_1 \geq 1 \\ A_{c2}(m_1, m_2) > A_{o2}(m_1, m_2 - 1),\ m_2 \geq 1 \end{array}\right\} \text{(internal stability)} \qquad (8)$$

$$\left.\begin{array}{l} A_{o1}(m_1, m_2) \geq A_{c1}(m_1 + 1, m_2),\ n_1 \geq 1 \\ A_{o2}(m_1, m_2) \geq A_{c2}(m_1, m_2 + 1),\ n_2 \geq 1 \end{array}\right\} \text{(external stability).} \qquad (9)$$

6.1 Simultaneous moves

In the simultaneous moves case, for most parameter values, these conditions are hard to meet. Like in the case of symmetric players, coalitions of more than two players cannot form. Moreover, the values of the parameters have to be high enough in order for a coalition of two members to form. Such a coalition can be homogeneous (two players of the same type) or heterogeneous. The main results are summarized in the following propositions.

Proposition 3 *: In the Great Fish War model with two types of asymmetric players, for any choice of strategic weights, the maximum stable* *coalition is of size 2 when outsiders and insiders announce their strategies simultaneously.*

Proof. See Appendix 8.2. ■

One obvious consequence of Proposition 3 is that any coalition of two members is externally stable. Moreover, it is easy to show that such a coalition is also profitable.

Proposition 4 *In the simultaneous moves case, if a partial coalition is stable, then it is also profitable.*

Proof. It suffices to note that (6) implies

$$A_{c1}^S(m_1, m_2) > A_{o1}^S(m_1 - 1, m_2) > A_1^N,\ m_1 \geq 1$$
$$A_{c2}^S(m_1, m_2) > A_{o2}^S(m_1, m_2 - 1) > A_2^N,\ m_2 \geq 1.$$

■

Therefore the internal stability conditions (8) are sufficient to verify the stability of a coalition of two members in the simultaneous moves case. Moreover, (6) also implies that stable coalitions of two members Pareto dominate the non-cooperative equilibrium. As we shall see with the two following propositions, size-two coalitions can form; however, as is the case for symmetric players, this only happen for extreme values of parameters.

Proposition 5 *: In the Great Fish War model with two types of asymmetric players, a stable homogeneous coalition of size 2 of players of type 1 can form if*

$$\beta_2 > \frac{\log 2}{1 - \log 2} n_2$$

and β_1 is sufficiently high.

Proof. see Appendix 8.3. ■

Proposition 6 *: In the Great Fish War model with two types of asymmetric players where $\beta_1 < \beta_2$, a stable heterogeneous coalition of size 2 can form if $\frac{\gamma_2}{\gamma_1} < e - 1$, and if β_1 and β_2 are sufficiently close and sufficiently high.*

Proof. see Appendix 8.4. ■

It is worthwhile noticing that one of the conditions for the existence of a heterogeneous stable coalition is that the players be relatively similar. Table 1 presents some illustrative results with 8 asymmetric players.

Table 1: Minimum parameter values ($\alpha\delta_1$) for the formation of coalitions when $M = 8$ and $\alpha\delta_2 = 0.95$

Composition / Possible coalitions	(2,6)	(4,4)	(6,2)
(2,0)	0	0,8983	0,9231
(2,0)-(0,2)	0,8972	0,9228	0,9317
(2,0)-(0,2)-$(1,1)^{pr}$	0,9412	0,9425	0,9435
(2,0)-(0,2)-$(1,1)^{eq}$	0,9465	0,9468	0,9469

Recall (Kwon, 2006) that, when players are identical, the threshold value for a stable coalition of size-two to exist is $\alpha\delta$ =0.9318 for $M = 8$. The results in Table 1 are obtained by fixing the parameter values for the most patient players at $\alpha\delta_2$ =0.95 and finding the minimum value for $\alpha\delta_1$ so that coalitions of various types can form. Columns refer to the composition of the population of players, that is, $(m_1 + n_1, m_2 + n_2)$. Lines refer to the possible coalitions, that is, homogeneous coalitions of impatient players (2,0), of patient players (0,2) or heterogeneous coalitions containing one patient and one impatient player (1,1). In that last case, the way the total catch is allocated to the players in the coalition matters, so that we report results for both the case where shares are inversely proportional to the players' bionomic multipliers $(1,1)^{pr}$ and shares are equal $(1,1)^{eq}$.

One can observe from these illustrative results that the parameter values are increasing horizontally and vertically: For all population distributions, it is easier to form coalitions of impatient players than of patient players, than heterogeneous coalitions. Moreover, it is easier to form stable heterogeneous coalitions when the strategic weights are proportional than when they are equal. On the other hand, we see that it is easier to form stable coalitions in populations where the number of impatient players is smaller. Indeed, if $\alpha\delta_2$ is high enough and there are only two impatient players in the population, then these two can form a stable coalition for any value of $\alpha\delta_1$.

6.2 Partial coordination with first mover advantage

When the coalition members have a first mover advantage, it is no longer the case that outsiders necessarily have higher payoffs than coalition members having the same characteristics. For that reason, stable coalitions are easier to obtain, and can involve more than two members. On the other hand, it is no longer the case that stable coalitions are necessarily profitable. This means that three conditions have to be verified for the two types of players, that is, profitability, internal stability, and external stability. To analyze how partial coordination can be achieved in that case, we study the behavior of the following functions for $i = 1, 2$:

$$
\begin{aligned}
p_i(m_1, m_2) = {} & \log \frac{\gamma_i}{G_m} \frac{1}{B_m + 1} + \beta_i \log \frac{B_m}{(B_m + 1)(b_n + 1)} \\
& - \log \frac{1}{\beta_i (b+1)} - \beta_i \log \frac{1}{(b+1)} \\
\psi_i(m_1, m_2) = {} & \log \frac{\gamma_i}{G_m} \frac{1}{B_m + 1} + \beta_i \log \frac{B_m}{(B_m + 1)(b_n + 1)} \\
& - \log \frac{B_{m-i}}{\beta_i (B_{m-i} + 1)(b_{n+i} + 1)} - \beta_i \log \frac{B_{m-i}}{(B_{m-i} + 1)(b_{n+i} + 1)} \\
\xi_i(m_1, m_2) = {} & \log \left(\frac{1}{\beta_i} \frac{B_m}{(B_m + 1)(b_n + 1)} \right) + \beta_i \log \left(\frac{B_m}{(B_m + 1)(b_n + 1)} \right) \\
& - \log \left(\frac{\gamma_i}{G_{m+i}} \frac{1}{B_{m+i} + 1} \right) - \beta_i \log \left(\frac{B_{m+i}}{(B_{m+i} + 1)(b_{n-i} + 1)} \right),
\end{aligned}
$$

where the (literal) subscripts $m \pm i$ and $n \pm i$ for G, B or b indicate that the sums are to be performed on the m first or n last players, to which player i is added or removed. Functions p_i, ψ_i and ξ_i take positive values when the profitability, internal stability and external stability conditions respectively are satisfied for a player of type i.

Proposition 7 *When players are divided into two groups characterized by their discount factor and strategic importance, there exists a vector* (γ_1, γ_2) *such that profitability of a partial coalition with first mover advantage is satisfied for any value of the fish stock.*

Proof. The proof is similar to the proof of Proposition 2. To see that $\gamma_1 = 1$ gives players of Type 1 a strictly higher payoff than what they would get in the competitive case:

$$
\max_X \left\{ \log X + \delta_1 A + \beta_1 \log(s - X - \frac{b_n}{b_n + 1}(s - X)) \right\} \geq A + (1 + \beta_i) \log s
$$

where

$$
(1 - \delta_1) A = \log \left(\frac{b - b_n}{b + 1} \right) + \beta_1 \log \left(\frac{1}{b + 1} \right)
$$

corresponding to a total catch of $s \frac{b - b_n}{b+1}$ by the members of the coalition and $s \frac{b_n}{b+1}$ by the outsiders. Using $b - b_n = b_m > \frac{1}{\beta_1}$ for $m_1 + m_2 \geq 2$ yields:

$$
\begin{aligned}
\log \left(\frac{b - b_n}{b + 1} \right) + \beta_1 \log \left(\frac{1}{b + 1} \right) & > \log \frac{1}{\beta_1 (b + 1)} + \beta_1 \log \left(\frac{1}{b + 1} \right) \\
& = (1 - \delta_1) A_1^N.
\end{aligned}
$$

As a consequence, it is always possible to find a weight vector $(\gamma_1^*, 1-\gamma_1^*)$ giving players of Type 1 in the coalition exactly their non-cooperative payoffs. The rest of the proof is the same as in Proposition 2 and is omitted. ■

The strict inequalities in the proof of Proposition 7 indicate that, as it is the case for the fully cooperative solution, there are in fact an infinite number of strategic weight vectors achieving profitability of a partial coalition. On the other hand, stability conditions do involve the strategic weight vector, so that there is not much that can be done analytically or even numerically to characterize sustainable (both stable and profitable) coalitions. In the sequel, we consider the equal weights and proportional weights scenarios, and investigate under what conditions these choices of weights give rise to sustainable coalitions.

Figure 7 graphs the stability functions for a representative case with ten players of each type, where Type 1 corresponds to impatient players ($\delta_1 = 0.85$) and Type 2 to patient players ($\delta_2 = 0.99$), and when strategic weights are inversely proportional to the players' bionomic multiplier. The general behavior of these functions is robust to the weight scenario choice and to changes in the values of the parameters, except for very extreme cases. Internal stability functions are generally decreasing in m_1 and m_2, and external stability functions are generally increasing in m_1 and m_2, for both types of players. Stability conditions depend both on the number of members and the number of outsiders, so that coalitions with a large membership with respect to the total number of players are externally stable, while coalitions with a small membership with respect to the total number of players are internally stable. Decreasing α, δ_1 or δ_2 shifts the internal stability functions upwards and the external stability downwards, therefore increasing the size of stable coalitions. Contrary to the symmetric case (see Kwon 2006), many stable configurations exist. For example, in the case represented in Figure 7, coalitions $(m_1, m_2) = (0,9)$, $(1,8)$, $(2,7)$, $(3,6)$, $(5,3)$, $(6,2)$, $(7,1)$ and $(8,0)$ are all stable (total number of coalition members can be 8 or 9).

However, stable coalitions are not necessarily profitable. Figure 8 illustrates the behavior of the profitability function. For both types of players, profitability is U-shaped, increasing with the number of coalition members if that number is large enough, where the threshold value for the profitability to be increasing with membership is decreasing with the number of members of the same type, and increasing with the number of players of the other type. Typically, for a given total number of members in a stable coalition, relatively large coalitions and relatively small coalitions are profitable; when the stable coalitions are of intermediary size, then one of the players' type is in minority. For instance,

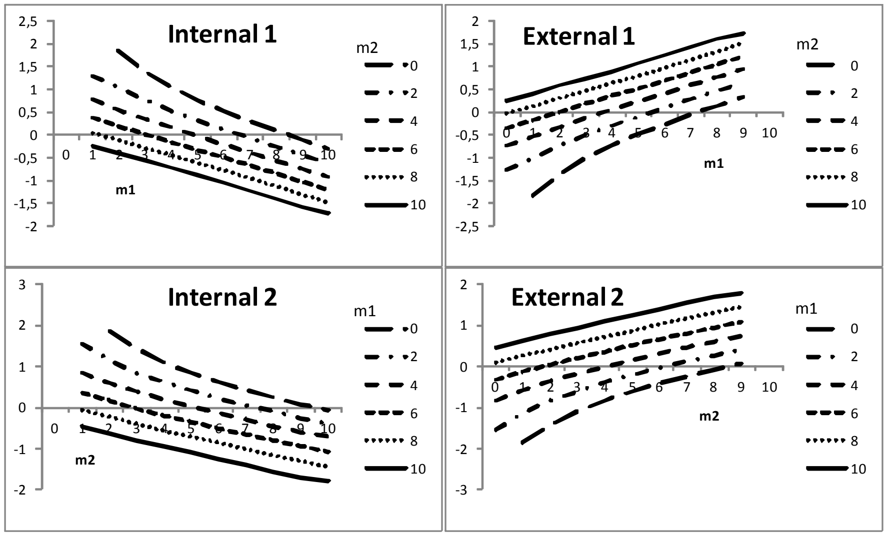

Figure 7: Stability as a function of the number of coalition members of each type. Parameter values are $M_1 = M_2 = 10$, $\alpha = 0.7$, $\delta_1 = 0.85$, $\delta_2 = 0.99$, $\gamma_i = \frac{1}{\beta_i}$. Coalitions are stable when functions take positive values.

in the case represented in Figures 7 and 8, the sustainable coalitions are $(m_1, m_2) = (0, 9)$, $(7, 1)$ and $(8, 0)$. As expected, profitability is harder to attain with equal strategic weights. The same example with equal strategic weights only has one sustainable coalition $(0, 9)$, composed of patient players only, and for the same set of parameters except for $\alpha = 0.5$, there exist no sustainable coalitions using equal strategic weights.

As a last remark, notice that in most situations, sustainable coalitions with first mover advantage are Pareto-improving, that is, outsiders to the coalition also have payoffs that are higher than in the non-cooperative case. To see why this is the case, observe that, by external stability,

$$A_{o1}^F(m_1, m_2) > A_{c1}^F(m_1 + 1, m_2).$$

If the total number $m_1 + m_2$ is high enough, the payoff of coalition members is increasing in $m_1 + m_2$, and

$$A_{c1}^F(m_1 + 1, m_2) > A_{c1}^F(m_1, m_2) > A_1^N$$

since the coalition is profitable.

7 Conclusion

To our knowledge, this paper is the first one to derive analytical results for the asymmetric case of the coalition versus fringe Great Fish War

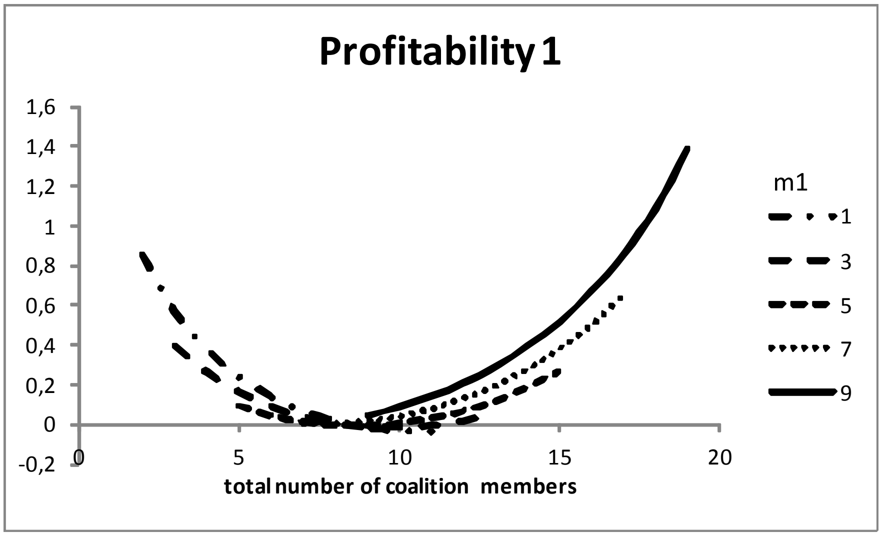

Figure 8: Profitability for players of Type 1 as a function of the total number of coalition members, for various values of m_1. Parameter values are as in Figure 7.

model. We find that the equilibrium fishing rule of each player (playing cooperatively or not) is a linear function of the current fish stock. We also show that, when players have different time preferences, it is still possible to find a set of weights so that the grand coalition is profitable. When players' preferences are very different, weights need to give more importance to the impatient players to preserve profitability. Weights that are proportional the inverse bionomic multipliers of the players achieve profitability in most cases with realistic parameter values. This choice of strategic weights is equivalent to allowing players in a coalition to share the total catch in the same proportion as they would do if they played non-cooperatively. If non-cooperative harvesting levels are used as an indication of players' discount factors, such a sharing mechanism is easy to implement and to justify in a coalition, and has a nice interpretation in terms of fishing capacity, where players joining a coalition do not need to drastically change their relative harvesting policy.

When players are divided into two broad categories with similar discount rates and strategic importance, we also study the stability and profitability of partial coalitions under two assumptions about coalition leadership. In the case where the coalition is not a dominant player and harvesting decisions of all players are simultaneous, the model predicts up to three possible stable coalitions of maximum size two – two homogeneous and one heterogeneous – but for relatively extreme parameter values, requiring the resource to be almost non-renewable. On the other hand, when cooperating players are given a first-mover advantage, as it is the case in many real situations, then larger-sized coalitions can form. Numerical experiments show that, in the range of reasonable parameter values, and when strategic weights are proportional to the players' inverse bionomic multipliers, there exist various configurations leading to sustainable coalitions that are also Pareto-improving.

8 Appendix

8.1 Equilibrium solutions

8.1.1 The non-cooperative case

Assuming that each player's value function takes the log-linear form

$$V_i^N(s) = A_i^N + (1+\beta_i)\log s,\ \ i = 1, ..., M,$$

then, if the total catch of the other players is X, Player i's optimization problem is written

$$\begin{aligned} &\max_{x_i} \left\{ \log(x_i) + \delta_i \left(A_i^N + (1+\beta_i) \log (s - X - x_i)^\alpha \right) \right\} \\ =&\max_{x_i} \left\{ \log(x_i) + \delta_i A_i^N + \beta_i \log (s - X - x_i) \right\}. \end{aligned}$$

First order conditions for interior solution for each player lead to the system:

$$\frac{1}{x_i} - \frac{\beta_i}{s - \sum_{j=1}^n x_j} = 0, \quad i = 1, \ldots, n$$

which is equivalent to the linear system:

$$\beta_i x_i + \sum_{j=1}^{n} x_j = s, \quad i = 1, ..., n. \tag{10}$$

Denoting $b \equiv \sum_{i=1}^M \beta_i^{-1}$, it is easy to check that $x_i = \frac{1}{\beta_i(1+b)} s$ is the unique solution to (10). As a consequence, the equilibrium fishing rule and residual stock are both linear in s, verifying the conjecture about the value function using Proposition 1, and the results follow.

To show that players with a lower discount rate have a smaller non-cooperative payoff than players with a higher discount rate, assume that $\beta_1 > \beta_2$ and consider the difference $V_1^N(s) - V_2^N(s)$:

$$\begin{aligned} (1+\beta_1)\log s - (1+\beta_2)\log s &= (\beta_1 - \beta_2)\log s \leq 0 \text{ since } s \leq 1 \\ A_1^N - A_2^N &= \log \frac{\beta_2}{\beta_1} + \left(\log \frac{1}{b+1} \right) (\beta_1 - \beta_2) < 0. \end{aligned}$$

8.1.2 The cooperative case

Assuming that the value function of each player when using the joint optimal strategy takes the log-linear form:

$$V_i^C(s) = A_i^C + (1+\beta_i)\log s,$$

then the optimization problem of the coalition is written:

$$\max_{X,\lambda}\left\{\sum_{i=1}^{M}\gamma_i\left(\log\lambda_i X + \delta_i A_i^C + \beta_i\log(s-X)\right)\right\}$$

where X is the total catch of the coalition and where the vector $\lambda \in \mathbb{R}^M$ satisfying $\lambda_i \geq 0$ and $\sum_{i=1}^{M}\lambda_i = 1$ indicates the way this catch is shared among members. Denoting $G \equiv \sum_{i=1}^{M}\gamma_i$ and $B \equiv \frac{\sum_{i=1}^{M}\gamma_i\beta_i}{G}$,the first order condition for the choice of X is then

$$\frac{G}{X} - \frac{BG}{s-X} = 0$$

yielding $X = \frac{s}{B+1}$. The total catch (and hence the residual stock) is therefore linear in the stock, and independent of the sharing vector λ. Maximizing $\sum_{i=1}^{M}\gamma_i\log\lambda_i$ under the constraint that $\sum_{i=1}^{M}\lambda_i = 1$ yields $\lambda_i = \frac{\gamma_i}{G}$. We have shown that the optimal fishing strategies and the residual stock are linear in the stock, thus verifying the assumption on the form of the value function using 1, and the results follow.

To show that, if two players have the same strategic weight, then the payoff of the more patient player is lower, assume that $\beta_1 > \beta_2$, $\gamma_1 = \gamma_2$, and consider the difference $V_1^N(s) - V_2^N(s)$:

$$\begin{aligned}(1+\beta_1)\log s - (1+\beta_2)\log s &= (\beta_1-\beta_2)\log s \leq 0 \text{ since } s \leq 1\\ A_1^N - A_2^N &= \log\frac{\gamma_1}{\gamma_2} + (\beta_1-\beta_2)\log\frac{B}{B+1} < 0.\end{aligned}$$

8.1.3 Simultaneous moves

Recalling that $B_m b_m > 1$, residual stock is higher under partial cooperation with simultaneous moves than in the non-cooperative case:

$$\begin{aligned}q^S - q^N &= \frac{B_m}{B_m(b_n+1)+1} - \frac{1}{1+b_m+b_n}\\ &= \frac{B_m b_m - 1}{(B_m(b_n+1)+1)(b_m+b_n+1)} > 0.\end{aligned}$$

On the other hand, using $B_n b_n > 1$, residual stock is lower than in the cooperative case:

$$
\begin{aligned}
q^S - q^C &= \frac{B_m}{B_m (b_n + 1) + 1} - \frac{B_m G_m + B_n G_n}{B_m G_m + B_n G_n + G_{m+n}} \\
&= -\frac{(B_m G_n (B_n b_n - 1) + B_n G_n + B_m^2 G_m b_n)}{(B_m + B_m b_n + 1)(G_{m+n} + B_m G_m + B_n G_n)} < 0.
\end{aligned}
$$

To see that the payoff of an outsider to a partial coalition is larger than what she would get in the non-cooperative case, we compare A_i^S to A_i^N and notice that $q^S > q^N$ and

$$
l_i^S = \frac{1}{\beta_i} \frac{1}{1 + b_n} > \frac{1}{\beta_i} \frac{1}{1 + b_m + b_n} = l_i^N.
$$

8.1.4 First mover advantage

Knowing that the outsiders' best response to a total catch of X is the linear rule

$$
y_i = \frac{1}{\beta_i} \frac{s - X}{b_n + 1}, i = m + 1, ..., M
$$

and assuming that players' payoff functions are given by

$$
V_i^F(s) = A_i^F + (1 + \beta_i) \log s,
$$

the coalition solves

$$
\max_{X,\lambda} \left\{ \sum_{i=1}^{m} \gamma_i \left(\log \lambda_i X + \delta_i A_i^F + \beta_i \log(s - X - \frac{b_n}{b_n + 1}(s - X)) \right) \right\}.
$$

Differentiating with respect to X yields the first order condition

$$
\frac{\sum_{i=1}^m \gamma_i}{X} - \frac{\sum_{i=1}^m \gamma_i \beta_i}{s - X} = \frac{G_m}{X} - \frac{B_m}{s - X} = 0
$$

or $X = \frac{s}{B_m + 1}$. Moreover, maximizing $\sum_{i=1}^n \gamma_i \log \lambda_i$ under the constraint that $\sum_{i=1}^n \lambda_i = 1$ yields $\lambda_i = \frac{\gamma_i}{G_m}$. Optimal fishing strategies and residual stock are therefore linear in the stock, verifying the assumption on the form of the value function using Proposition 1, and the results follow.

Coalition members harvest more, and outsiders harvest less than they would have in the simultaneous moves case:

$$
\begin{aligned}
\frac{h_i^F}{h_i^S} &= \frac{\frac{\gamma_i}{G_m} \frac{1}{B_m + 1}}{\frac{\gamma_i}{G_m} \frac{1}{B_m (b_n + 1) + 1}} = \frac{B_m + 1 + B_m b_n}{B_m + 1} > 1, i = 1, ...m \\
\frac{h_i^F}{h_i^S} &= \frac{\frac{1}{\beta_i} \frac{B_m}{(B_m + 1)(b_n + 1)}}{\frac{1}{\beta_i} \frac{B_m}{B_m (b_n + 1) + 1}} = \frac{B_m + B_m b_n + 1}{B_m + B_m b_n + 1 + b_n} < 1, i = m + 1, ...M,
\end{aligned}
$$

while the residual stock is smaller than in the case of simultaneous moves:

$$\frac{q^F}{q^S} = \frac{\frac{B_m}{(B_m+1)(b_n+1)}}{\frac{B_m}{B_m(b_n+1)+1}} = \frac{B_m + B_m b_n + 1}{B_m + B_m b_n + 1 + b_n} < 1.$$

8.2 Stability of coalitions of size greater than 2

A coalition of m_1 members of type 1 and m_2 members of type 2 is internally stable if it satisfies

$$A_{c1}(m_1, m_2) > A_{o1}(m_1 - 1, m_2) \text{ and } A_{c2}(m_1, m_2) > A_{o2}(m_1, m_2 - 1).$$

Without loss of generality, assume that $m_1 \geq 1$ and $\gamma_1 = 1$. The internal stability condition for players of type 1 is then equivalent to

$$\log \frac{\beta_1}{B_m G} + (1 + \beta_1) \log \frac{B_m}{B_{m-1}} \frac{B_{m-1}(b_{n+1} + 1) + 1}{B_m (b_n + 1) + 1} > 0 \qquad (11)$$

where the (literal) subscripts $m-1$ and $n+1$ indicate that the sums are taken respectively over the first m players except Player 1 and the last n players with the addition of Player 1, $G = m_1 + m_2\gamma_2$, $B_m = \frac{m_1\beta_1 + m_2\gamma_2\beta_2}{G}$, $B_{m-1} = \frac{(m_1-1)\beta_1 + m_2\gamma_2\beta_2}{(m_1-1)+m_2\gamma_2}$, $b_n = \frac{n_1}{\beta_1} + \frac{n_2}{\beta_2}$, $b_{n+1} = \frac{n_1+1}{\beta_1} + \frac{n_2}{\beta_2}$.

Consider the function

$$g_1(n_1, n_2) = \frac{B_{m-1}\left(\frac{n_1+1}{\beta_1} + \frac{n_2}{\beta_2} + 1\right) + 1}{B_m \left(\frac{n_1}{\beta_1} + \frac{n_2}{\beta_2} + 1\right) + 1}.$$

It is easy to check that g_1 is decreasing in both n_1 and n_2 :

$$\frac{\partial g_1}{\partial n_1} = -\beta_2^2 \frac{G}{G-1} \times \frac{\beta_1 (m_1 - 1)(\beta_1 m_1 + 2\beta_2\gamma_2 m_2) + \beta_1^2 \gamma_2 m_2 + \beta_2^2 \gamma_2^2 m_2^2}{\left(\beta_1^2 m_1 (\beta_2 + n_2) + \beta_2^2 \gamma_2 m_2 (\beta_1 + n_1) + \beta_1\beta_2 (m_1 (n_1 + 1) + \gamma_2 m_2 (n_2 + 1))\right)^2}$$

$$\frac{\partial g_1}{\partial n_2} = -\beta_1\beta_2 \frac{G}{G-1} \times \frac{\beta_1 (m_1 - 1)(\beta_1 m_1 + 2\beta_2\gamma_2 m_2) + \beta_2^2 \gamma_2^2 m_2^2 + \beta_1^2 \gamma_2 m_2}{\left(\beta_1^2 m_1 (\beta_2 + n_2) + \beta_2^2 \gamma_2 m_2 (\beta_1 + n_1) + \beta_1\beta_2 (m_1 (n_1 + 1) + \gamma_2 m_2 (n_2 + 1))\right)^2},$$

so that

$$g_1(n_1, n_2) \leq \frac{B_{m-1}\left(\frac{1}{\beta_1} + 1\right) + 1}{B_m + 1}$$

and consequently

$$\begin{aligned}
&\log\frac{\beta_1}{B_m G}+(1+\beta_1)\log\frac{B_m}{B_{m-1}}\frac{B_{m-1}(b_{n+1}+1)+1}{B_m(b_n+1)+1}\\
\le&\log\frac{\beta_1}{B_m G}+(1+\beta_1)\log\frac{B_m}{B_{m-1}}\frac{B_{m-1}\left(\frac{1}{\beta_1}+1\right)+1}{B_m+1}. \qquad (12)
\end{aligned}$$

First assume that $m_2=0$, so that $G=m_1$ and $B_m=B_{m-1}=\beta_1$. We then have

$$\begin{aligned}
&\log\frac{\beta_1}{B_m G}+(1+\beta_1)\log\frac{B_m}{B_{m-1}}\frac{B_{m-1}\left(\frac{1}{\beta_1}+1\right)+1}{B_m+1}\\
=&-\log m_1+(1+\beta_1)\log\left(\frac{1}{\beta_1+1}+1\right)\\
<&-\log m_1+1
\end{aligned}$$

using $\log(y+1)<y$ for $y>0$. This last expression is negative for $m_1>2$. We have shown that homogeneous coalitions of more than 2 players are not stable.

Now assume that $m_1\ge 1$, $m_2\ge 1$, and $M\ge 3$.We first consider the special case where $\gamma_2=\frac{\beta_1}{\beta_2}$ (strategic weights are inversely proportional to players' bionomic multipliers), that is, $B_m=\frac{\beta_1 M}{G}$, $B_{m-1}=\beta_1\frac{M-1}{G-1}$, and we assume that $\gamma_2<1$. The stability condition for players of type 1 then becomes

$$\log\frac{1}{M}+(1+\beta_1)\log M\frac{M-2+\gamma_2 m_2+m_1+\beta_1(M-1)}{(M-1)(m_1+\gamma_2 m_2+M\beta_1)}>0.$$

It is easy to check that $\frac{M-2+\gamma_2 m_2+m_1+\beta_1(M-1)}{(M-1)(m_1+\gamma_2 m_2+M\beta_1)}$ is monotone in γ_2 : it is non-decreasing in γ_2 when $\beta_1\ge M-2$, and decreasing otherwise. If $\beta_1\ge M-2$, then using $\gamma_2<1$,

$$\begin{aligned}
&\log\frac{1}{M}+(1+\beta_1)\log M\frac{M-2+\gamma_2 m_2+m_1+\beta_1(M-1)}{(M-1)(m_1+\gamma_2 m_2+M\beta_1)}\\
\le&\log\frac{1}{M}+(1+\beta_1)\log\frac{\beta_1+2}{\beta_1+1}<\log\frac{1}{M}+1,
\end{aligned}$$

which is negative for $M\ge 3$. We conclude that the stability condition is not satisfied for Players of type 1 if $M-2\le\beta_1<\beta_2$.

On the other hand, if $\beta_1<M-2$ then, using $\gamma_2>0$,

$$\begin{aligned}
&(1+\beta_1)\log M\frac{M-2+\gamma_2 m_2+m_1+\beta_1(M-1)}{(M-1)(m_1+\gamma_2 m_2+M\beta_1)}\\
<&(1+\beta_1)\log M\frac{M-2+m_1+\beta_1(M-1)}{(M-1)(m_1+M\beta_1)}
\end{aligned}$$

where this last expression is decreasing in m_1 for $\beta_1 < M - 2$, so that we get finally

$$\begin{aligned} &\log \frac{1}{M} + (1+\beta_1) \log M \frac{M-2+m_1+\beta_1 (M-1)}{(M-1)(m_1+M\beta_1)} \\ <&\log \frac{1}{M} + (1+\beta_1) \log M \frac{\beta_1+1}{M\beta_1+1} \\ <&\log \frac{1}{M} + \log M = 0. \end{aligned}$$

We conclude that, in the proportional weight case, the stability condition is never satisfied for the less patient players when they are in a heterogeneous coalition of more than 2 players.

Now we consider the general weight case, assuming $\beta_1 < \gamma_2\beta_2$. Using a change of variable $A \equiv m_1 + \frac{\gamma_2\beta_2}{\beta_1} m_2 > M$, (12) can be written as a function of A, where $G = m_1 + m_2\gamma_2 \geq 1$, $B_m = A\frac{\beta_1}{G}$, $B_{m-1} = \beta_1 \frac{A-1}{G-1}$:

$$\begin{aligned} &\log \frac{\beta_1}{B_m G} + (1+\beta_1) \log \frac{B_m}{B_{m-1}} \frac{B_{m-1}\left(\frac{1}{\beta_1}+1\right)+1}{B_m+1} \\ =&\log \frac{1}{A} + (1+\beta_1) \log \frac{A(G+(A+\beta_1(A-1)-2))}{(G+A\beta_1)(A-1)}. \end{aligned}$$

Differentiating this last expression w.r.t. A yields

$$\begin{aligned} &-\frac{\beta_1^2 (A+G(2A-1)+A^2(A-3))+AG(G-1)}{A(A-1)(G+A\beta_1)(A+G-\beta_1+A\beta_1-2)} \\ &-\frac{\beta_1 (2A+A^2(G-1)+G(A-2)+A^2(A-3)+G^2)}{A(A-1)\left(\beta_1^2 A(A-1)+\beta_1(G(2A-1)+A(A-2))+G(A-2+G)\right)} \end{aligned}$$

which is negative for $A \geq 3$.We conclude that (12) is decreasing in β_2 and consequently since we assumed $\beta_2 > \frac{\beta_1}{\gamma_2}$,

$$\begin{aligned} &\log \frac{1}{A} + (1+\beta_1) \log \frac{A(G+(A+\beta_1(A-1)-2))}{(G+A\beta_1)(A-1)} \\ <&\log \frac{1}{M} + (1+\beta_1) \log M \frac{M-2+\gamma_2 m_2+m_1+\beta_1(M-1)}{(M-1)(m_1+\gamma_2 m_2+M\beta_1)} \end{aligned}$$

which we already showed to be negative for $M \geq 3$.

We finally conclude that the stability condition in a heterogeneous coalition of more than 2 players is never satisfied for players with the lowest discount rate.

8.3 Existence of homogeneous coalitions of size 2

Suppose without loss of generality that $\gamma_1 = 1$, $m_1 = 2$ and $m_2 = 0$, we want to find conditions so that (11) is satisfied where $G = 2$, $B_m = B_{m-1} = \beta_1$. Define the function

$$g_2(\beta_1) = (1+\beta_1)\log\frac{\beta_1\left(\frac{n_1+1}{\beta_1}+\frac{n_2}{\beta_2}+1\right)+1}{\beta_1\left(\frac{n_1}{\beta_1}+\frac{n_2}{\beta_2}+1\right)+1} - \log 2$$

and notice that $\lim_{\beta_1\to\infty} g_2 = \frac{\beta_2}{\beta_2+n_2} - \log 2$, which is positive if $\beta_2 > \frac{\log 2}{1-\log 2}n_2$. We conclude that the stability condition is satisfied if $\beta_2 > \frac{\log 2}{1-\log 2}n_2$ and $\beta_1$is sufficiently high. To see that this condition is necessary, assume $\beta_2 \leq \frac{\log 2}{1-\log 2}n_2$. We then have

$$\begin{aligned}
&(1+\beta_1)\log\frac{2\beta_2+\beta_1\beta_2+\beta_1 n_2+\beta_2 n_1}{\beta_2+\beta_1\beta_2+\beta_1 n_2+\beta_2 n_1} - \log 2\\
\leq &(1+\beta_1)\log\frac{2\beta_2+\beta_1\beta_2+\beta_1 n_2}{\beta_2+\beta_1\beta_2+\beta_1 n_2} - \log 2\\
\leq &(1+\beta_1)\log\frac{2\frac{\log 2}{1-\log 2}n_2+\beta_1\frac{\log 2}{1-\log 2}n_2+\beta_1 n_2}{\frac{\log 2}{1-\log 2}n_2+\beta_1\frac{\log 2}{1-\log 2}n_2+\beta_1 n_2} - \log 2\\
= &(1+\beta_1)\log\frac{(\beta_1+2\log 2)}{\beta_1+\log 2} - \log 2,
\end{aligned}$$

where

$$\begin{aligned}
&\frac{d}{d\beta_1}\left((1+\beta_1)\log\frac{(\beta_1+2\log 2)}{\beta_1+\log 2} - \log 2\right)\\
= &\log\frac{(\beta_1+2\log 2)}{\beta_1+\log 2} - (\log 2)\frac{\beta_1+1}{(\beta_1+2\log 2)(\beta_1+\log 2)}. \qquad (13)
\end{aligned}$$

If (13) is negative, then

$$(1+\beta_1)\log\frac{(\beta_1+2\log 2)}{\beta_1+\log 2} - \log 2 < \log 2 - \log 2 = 0.$$

If (13) is positive, then

$$\begin{aligned}
&(1+\beta_1)\log\frac{(\beta_1+2\log 2)}{\beta_1+\log 2} - \log 2\\
< &\lim_{\beta_1\to\infty}(1+\beta_1)\log\frac{(\beta_1+2\log 2)}{\beta_1+\log 2} - \log 2 = 0.
\end{aligned}$$

We conclude that the stability condition cannot be satisfied if $\beta_2 \leq \frac{\log 2}{1-\log 2}n_2$.

8.4 Existence of the heterogeneous coalition (1,1)

We now want to find sufficient conditions so that (11) is satisfied for both types of players, that is:

$$\log \frac{\beta_1}{B_m G} + (1+\beta_1)\log \frac{B_m}{\beta_2}\frac{\beta_2\left(b_n + \frac{1}{\beta_1} + 1\right) + 1}{B_m\left(b_n+1\right)+1} > 0 \text{ for Player 1}$$

$$\log \frac{\beta_2}{B_m G} + (1+\beta_2)\log \frac{B_m}{\beta_1}\frac{\beta_1\left(b_n + \frac{1}{\beta_2} + 1\right) + 1}{B_m\left(b_n+1\right)+1} > 0 \text{ for Player 2.}$$

Notice that

$$\begin{aligned}\frac{B_m}{\beta_2}\frac{\beta_2\left(b_n + \frac{1}{\beta_1} + 1\right) + 1}{B_m\left(b_n+1\right)+1} &= \frac{B_m}{\beta_1}\frac{\beta_1\left(b_n + \frac{1}{\beta_2} + 1\right) + 1}{B_m\left(b_n+1\right)+1} \\ &= \frac{B_m\left(\beta_1+\beta_2+\beta_1\beta_2+\beta_1\beta_2 b_n\right)}{\beta_1\beta_2\left(B_m+B_m b_n+1\right)} \\ &= \frac{\beta_1^2\gamma_1+\beta_2^2\gamma_2}{\beta_1\beta_2\left(\gamma_1+\gamma_2\right)\left(B_m+B_m b_n+1\right)}+1\end{aligned}$$

so that it suffices to check the stability condition for the less patient player, having the smallest multiplier β. Assume without loss of generality that $\beta_1 < \beta_2$ and $\gamma_1 = 1$, set $\frac{\beta_2}{\beta_1} = x > 1$ and define the function:

$$\begin{aligned}g_3(\beta_1, x) = {} & \log \frac{1}{x\gamma_2+1} \\ & + (1+\beta_1)\log \frac{\left(x\gamma_2+1\right)\left(x+n_2+x\beta_1+xn_1+1\right)}{x+n_2+x\beta_1+x\gamma_2+xn_1+x\gamma_2 n_2+x^2\beta_1\gamma_2+x^2\gamma_2 n_1};\end{aligned}$$

it is easy to check that this function is decreasing in x, and therefore its maximum value is attained at $x = 1$:

$$g_3(\beta_1, x) \leq \log \frac{1}{\gamma_2+1} + (1+\beta_1)\log \frac{\beta_1+n_1+n_2+2}{\beta_1+n_1+n_2+1}; \tag{14}$$

The second member of the bound in (14) is increasing in $\beta_{1,}$ with

$$\lim_{\beta_{1\to\infty}} (1+\beta_1)\log \frac{\beta_1+n_1+n_2+2}{\beta_1+n_1+n_2+1} = 1,$$

so that

$$g_3(\beta_1, x) \leq \log \frac{1}{\gamma_2+1} + 1,$$

and a heterogeneous coalition cannot be stable if $\log\left(\gamma_2+1\right) \geq 1$.

If γ_2 does not depend on the relative values of the players' bionomic factors and $\log(\gamma_2 + 1) < 1$, then since the value of $g_3(\beta_1, x)$ is strictly positive when $x \to 1$ and $\beta_1 \to \infty$, a heterogeneous coalition is stable if β_1 and β_2 are sufficiently close and sufficiently high. This is the case when, for instance, players have equal strategic weights.

In the proportional weights case, $\gamma_2 = \frac{1}{x}$. The stability condition is then related to the sign of function g_4 defined by:

$$g_4(\beta_1, x) = -\log 2 + (1+\beta_1)\log\left(2\frac{x + n_2 + x\beta_1 + xn_1 + 1}{x + 2n_2 + 2x\beta_1 + 2xn_1 + 1}\right).$$

Differentiating g_4 with respect to x yields:

$$-(\beta_1 + 1)\frac{\beta_1 + n_1 - n_2}{(x + n_2 + x\beta_1 + xn_1 + 1)(x + 2n_2 + 2x\beta_1 + 2xn_1 + 1)}$$

and two possible cases can arise.

If $\beta_1 > n_2 - n_1$, g_4 is decreasing in x and its maximum value is attained at $x = 1$:

$$g_4(\beta_1, x) < (1+\beta_1)\log\left(\frac{\beta_1 + n_1 + n_2 + 2}{\beta_1 + n_1 + n_2 + 1}\right) - \log 2,$$

which is strictly positive when β_1 is high enough,

$$\lim_{\beta_1 \to \infty} (1+\beta_1)\log\left(\frac{\beta_1 + n_1 + n_2 + 2}{\beta_1 + n_1 + n_2 + 1}\right) - \log 2 = 1 - \log 2 > 0.$$

If on the other hand $\beta_1 \leq n_2 - n_1$, g_4 is non decreasing in x, so that

$$\begin{aligned} g_4(\beta_1, x) &< -\log 2 + (1+\beta_1)\log\left(\frac{2\beta_1 + 2n_1 + 2}{2\beta_1 + 2n_1 + 1}\right) \\ &< -\log 2 + (1+\beta_1)\log\left(\frac{2\beta_1 + 2}{2\beta_1 + 1}\right) < 0 \end{aligned}$$

and a heterogeneous coalition cannot be stable. We conclude that when weights are inversely proportional to the bionomic multipliers, a heterogeneous coalition can form if β_1 and β_2 are sufficiently close and sufficiently high.

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